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How do Beveridge and Phillips curves in the Euro Area behave under the stress of the World Economic Crisis?*

by Friedrich L. Sell† and David C. Reinisch‡

Abstract

In this paper, the authors present a new concept of the "modified output gap" based on the New Keynesian Phillips curve and on the Beveridge curve. In the first part of the paper, both mentioned curves are derived analytically. In doing so, we identify key parameters for the shift of the Beveridge curve (up- or downwards) and prove that - fulfilling a minimum of assumptions - the New Keynesian Phillips curve is (also) a falling convex relationship between the inflation rate and the unemployment rate in the tradition of Phillips (1958). Inserting the Phillips curve into the Beveridge curve reveals the explicit positive relationship between the vacancy ratio and the inflation rate. In the second part of the paper, we put all three relationships under an empirical "stress test" using panel data from eleven of the EA 12 countries for three different samples during the world economic crisis: In all cases, the parameter estimates confirm the presumed existence of the three functions.

JEL classification: J63, J64, J38, E24
key words: Labor Market, Phillips and Beveridge curves, policy ineffectiveness, world economic crisis and output gap

*The authors would like to thank Rolf Scheufele, Beate Sauer and Thomas Werner for their helpful hints. This working paper is an elaborate version of a paper accepted for publishing in the *International Labour Review*. The main extension here is a detailed theory section which explains the causalities of the Beveridge and the Phillips curve on the one hand and combines these two concepts to a new economically interpretable relationship between the inflation rate and the vacancy ratio on the other hand: the so called "modified output gap" .

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1 Introduction

It is an undisputed fact that the discipline of economics is under fire ever since the collapse of Lehman Brothers in fall of 2008; what has been labeled up to then "mainstream economics" is criticized forcefully for its incapability to detect and to predict correctly key elements of the financial and of the concomitant real economic crisis. In light of this development, we will raise in this paper, so to say, the inverse question: How did more or less "traditional" (and to some members of the scientific community almost obsolete) tools of macroeconomics, such as very much the Beveridge and less so the Phillips curve perform empirically during the recent world economic crisis?

In order to conduct the appropriate empirical analysis, a number of theoretical considerations have to be accomplished: we first show the convexity of the theoretical Beveridge curve and identify parameters responsible for shifts of the function. We then prove that the "New Keynesian Phillips curve" can be transformed – with almost no critical assumptions to be made – into a traditional "trade off relationship" between unemployment and the inflation rate. Thirdly, we combine these two tools for the setup of a "modified output-gap" which relates different inflation rates to associated vacancy ratios: The higher the prevalent vacancy ratios – which signal a significant mismatch in the labour market – the higher the observed inflation rates, ceteris paribus. A high vacancy ratio symbols the lost benefit from the capability to fill vacant jobs with appropriate candidates and to be able to expand the production of goods in reaction to an increased demand without creating an inflationary pressure.

Putting this framework into the setting of the world economic crisis means the following: a collapsing private demand leads to higher unemployment, to lower vacancy ratios and to lower inflation, too. Starting from the premise that central banks around the world will by all means try to avoid deflation implies that inflation expectations can be anchored at values close to price stability. In other words: the economies affected will most likely move down alongside their individual Phillips curves rather than experience inward shifts of these Phillips curves. The appropriate empirical test for this view is to find a "stable" Phillips curve in the data.

Where do the data come from? We gathered quarterly observations – from EUROSTAT – for the countries which build the Euro Area. The rationale behind is threefold: in the first place, according to the theory of endogenous criteria of optimal currency areas, members of a currency union tend to synchronize their business cycles. Secondly, these countries are affected symmetrically by exchange rate changes vis-à-vis the Euro. Thirdly, these countries share a common central bank which has been fighting almost from the beginning the severe financial and economic crisis by liquidity enhancing instruments together with a low interest rate policy.

The paper is organized as follows: In the next section, we develop a simple model which integrates the Beveridge curve and the (adjusted) New Keynesian Phillips curve and derives from these two concepts the “modified output gap”. Thereafter, we test empirically these three schedules with data from Euro Area countries. At the end of the paper, we derive some policy implications in this respect.

2 An integrated model for the Beveridge and for the Phillips curve

2.1 Derivation of the Beveridge curve

After the pioneering work of Dow/Dicks-Mireaux (1958), both the theoretical underpinning and the empirical relevance of the Beveridge curve continue to be addressed directly (see Wall/Zoega (2002); Shimer (2005)) or at least indirectly (see Hall (2005)) by well known economists. The concept of the Beveridge curve (see Cahuc/Zylberberg (2004)) is based on the presumed existence of a general mediation function on the labour market. It is assumed that there is a positive relationship between the likelihood to be located at a job on the one hand and the number of current vacant positions in firms. The probability that an unemployed person will re-enter the formal labour market again after one period of unemployment (η), is approximated by the following mediation function

$$\eta = \mathrm{g}(\theta) \quad \text{with} \quad \theta = \frac{V}{U}, U = \frac{V}{\theta}. \tag{1}$$

V denotes the number of vacant positions and U the number of dismissed persons. The function $g(\theta)$ is characterized by decreasing marginal returns: $g' > 0, g'' < 0$. In the following, we will make use of a rather simple mediation function which has the required properties (see Landmann/Jerger (1999), pp. 54-58)[1]:

$$\eta = \eta_0 \cdot \theta^{0,5}. \tag{2}$$

Taking the existing flow variables on the labour market into account, one can define the change in the number of employed persons as the difference between the quantity of former unemployed persons who have found a new job in the same period (ηU) on the one hand and the number of those individuals who have lost their job in the corresponding period (sA) on the other hand. The product sA is equal to the probability to lose one's actual job (s) times the number of employed people (A). Therefore, the change of employment within a period t denotes:

$$\Delta A_t = \eta U - sA. \tag{3}$$

Inserting (2) into (3) gives:

$$\Delta A_t = \eta U - sA = \eta_0 \underbrace{V^{0,5} U^{-0,5}}_{\theta^{0,5}} \cdot U - sA = \eta_0 (V \cdot U)^{0,5} - sA. \tag{4}$$

In the case of flow equilibrium ($\Delta A_t = 0$), it is possible to calculate the equilibrium size of unemployment:

$$\Delta A_t = 0 \rightarrow sA = \eta_0 (V \cdot U)^{0,5}. \tag{5}$$

This equation leads us directly to the Beveridge curve.[2]

$$v = \left(\frac{s}{\eta_0}\right)^2 \cdot \left[\frac{1}{u} - 2 + u\right]. \tag{6}$$

The curvature of this function can be easily assessed by taking the first and second derivatives of v with respect to u. The first derivative reads:

$$\frac{\partial v}{\partial u} = \left(\frac{s}{\eta_0}\right)^2 \cdot \left[-\frac{1}{u^2} + 1\right] < 0, \quad \text{for} \quad u < 1. \tag{7}$$

[1]As an alternative, one may use directly a more sophisticated matching function, which, however, leads to the same convex Beveridge curvature (compare Warren (1983), Blanchard /Diamond (1989); De Francesco (1999); Wall/Zoega (2002).

[2]Notice that, as an alternative, one may first specify a matching function with constant returns to scale, define an equilibrium, where the number of separations equals the number of matches, assume a fixed separation rate and intercept to achieve a negative relationship between the unemployment and the vacancy rate (Wall/Zoega (2002), p. 259). For a complete derivation, please see part 1 of the mathematical annex.

As the unemployment rate is only defined for values between 0 and 1, the first derivative $\frac{\partial v}{\partial u}$ is always less than 0, i. e., the function has a negative slope. The second derivative discloses how skewed the function is:

$$\frac{\partial^2 v}{\partial u^2} = \left(\frac{s}{\eta_0}\right)^2 \cdot \left[-\frac{-2}{u^3}\right] = \left(\frac{s}{\eta_0}\right)^2 \cdot \left[\frac{2}{u^3}\right] > 0. \tag{8}$$

As s, η and u always exceed 0, the second derivative, according to (8), will always be positive. Hence the implied Beveridge curve describes a negative convex relationship between the unemployment rate u and the vacancy ratio v! From (6), we can see that the term $\left(\frac{s}{\eta_0}\right)$ seems to be decisive for the position of any Beveridge curve. A simple procedure to analyze the impact of this term is to set $u = v$, which is equivalent to consider all points of different Beveridge curves which are located on the 45 angle line. This leads us to

$$v = \left(\frac{s}{\eta_0}\right)^2 \cdot \frac{(1-v)^2}{v}; \quad v^2 = \left(\frac{s}{\eta_0}\right)^2 \cdot (1-v)^2 \tag{9}$$

or

taking roots on both sides of the equation:

$$v = \left(\frac{s}{\eta_0}\right) \cdot (1-v). \tag{10}$$

If we now solve equation (10) for the variable v, we get a new equation which enables us to determine the factors responsible for such points on the Beveridge curve which, at the same time, belong to the 45 angle line

$$v = \frac{\frac{s}{\eta_0}}{1 + \frac{s}{\eta_0}} \tag{11}$$

Of course, equations (10) / (11) reproduce only one of the two solutions a quadratic equation such as (9) usually has. But, the second solution one can get from (9) is

$$v = -\frac{s}{\eta_0}/1 - \frac{s}{\eta_0}. \tag{12}$$

and it can be excluded as irrelevant right away, given that the solutions for v implied do not satisfy the interval of permitted values for the vacancy ratio ([0,1]).

From the interpretation of equation (11), it seems to be quite easy to assess the most likely consequences structural policies on the labour market will have on the position of the Beveridge curve. Any proposal/policy instrument which either tends to increase the probability to be fired s and/or to reduce the likelihood for getting a job η_0, lowers the efficiency of mediation on the labour market, and, at the same time, increases the relevant values of u and v, ceteris paribus.

2.2 Derivation of a modified New Keynesian Phillips curve

As in Jordi Galís seminal paper (2000, p. 8) the dynamics of inflation according to the new Phillips curve have real marginal costs as the main driving force:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \hat{mc}_t^r. \tag{13}$$

Supposing the existence of a wedge between the actual output level and the equilibrium output level – this phenomenon could mirror the fact that firms are unable to adjust their prices optimally every period (ibid., p. 8) – , we relate, following Galí (2000, p. 18), Zhang et al. (2006, p. 5) and Kuttner/Robinson (2008, p. 2), real marginal costs $\hat{mc}_t^r$ to a measure of the output gap ($\hat{y}_t = y_t - \tilde{y}_t$). We further assume that the costs of production behave pro-cyclically: when actual production exceeds potential output, the marginal costs rise more than output prices do:

$$\hat{mc}_t^r = \varphi \hat{y}_t; \quad \varphi > 0. \tag{14}$$

We now substitute the expression $E_t \pi_{t+1}$ by π_t^e (following Galí/Gertler (1999) as well as Galí et. al. (2001)) and define $\varphi\lambda = \kappa$. It then follows (see also Rumler/Valderrama (2008):

$$\pi_t = \beta \pi_t^e + \kappa \hat{y} t. \tag{15}$$

If stochastic shocks are taken into account, the equation now reads:

$$\pi_t = \beta \pi_t^e + \kappa \hat{y} t + \varepsilon_t. \tag{16}$$

A variant of (16) is a sort of „hybrid“ equation where past inflation enters the r.h.s. Now, inflationary expectations π_t^e and past inflation π_{t-1} are weighted by the parameters α and $(1 - \alpha)$, and respectively. It follows:

$$\pi_t = (1 - \alpha)\pi_t^e + \alpha \pi_{t-1} + \kappa \hat{y}_t + \varepsilon_t \tag{17}$$

We now introduce a number of widely accepted definitions in labour market economics; we start with the actual unemployment rate:

$$u_t = \frac{A_t^{fe} - A_t}{A_t^{fe}}; \quad \text{A = number of employees, fe = full employment}$$

$$\tilde{u}_t = \frac{A_t^{fe} - \tilde{A}_t}{A_t^{fe}}; \quad \tilde{A} = \text{number of employees in equilibrium}$$

$$\tilde{u}_t - u_t = \frac{A_t^{fe} - \tilde{A}_t - A_t^{fe} + A_t}{A_t^{fe}}; = \frac{A_t - \tilde{A}_t}{A_t^{fe}}; \quad \tilde{u} = \text{equilibrium rate of unemployment.}$$

The respective output can be gained by a simple calculation; actual output, y_t, is the product of labour productivity a_t and the total of hours worked H_t:

$$y_t = a_t H_t; \;\; a_t = \frac{y_t}{H_t} = a. \tag{18}$$

Hours worked, in turn, can be calculated as the number of employed persons, A_t, times hours worked per employee, $H_t/A_t(h_t)$:

$$H_t = A_t \frac{H_t}{A_t} = A_t h(t) = A_t h. \tag{19}$$

Hence,

$$y_t = ahA_t; \;\; \tilde{y}_t = ah\tilde{A}_t. \tag{20}$$

With the help of equation (20), we can rewrite the definition of the output gap:

$$(y_t - \tilde{y}_t) = ah[A_t - \tilde{A}_t] = A_t^{fe} \cdot ah(\tilde{u}_t - u_t) = \gamma(\tilde{u}_t - u_t); with \;\; \gamma = A^{fe}ah. \tag{21}$$

Inserting this finding into equation (16) gives:

$$\pi_t = \beta\pi_t^e + \kappa\gamma(\tilde{u}_t - u_t) + \varepsilon_t. \tag{22}$$

With this formulation, we sort of "replicate" earlier results achieved already by Mankiw (2001) and slightly modified by Tavlas/Swamy (2007). However, and different from these mentioned authors, our derivation of (22) depends on a minimum of assumptions, if any. Making use of (17), we get the following equation (23) which is identical to the results achieved in Galí (2003) and in Walsh (2003):

$$\pi_t = (1 - \varphi)\pi_t^e + \varphi\pi_{t-1} + \kappa\gamma(\tilde{u}_t - u_t) + \varepsilon_t. \tag{23}$$

In this formulation, we can see "reappear" as in Ravenna/Walsh (2007, pp. 11-15) the famous "trade off" between the rate of inflation and the rate of unemployment. However, when closely inspected (see ibid, p. 12), their final equation (22) brings in much more additional variables and, more than anything else, tries to give an intense explanation of inflationary processes, including, among other things, the real costs of capital.

2.3 The complete model: Derivation of the modified output gap (MOG)

As we intend to show in the following, the Beveridge curve can be combined with the new Phillips curve. Notice that this idea goes back to Blanchard/Diamond (1989)(p.

51): “The model suggests both an integrated way of thinking about the Phillips curve and the Beveridge curve.” The result of this combination is a new economic function with a positive relationship between the vacancy ratio on one hand and the inflation rate on the other hand: $\pi_t = f(\nu_t); \frac{\partial \pi_t}{\partial \nu_t} > 0$. For a full derivation of this relationship, see the mathematical annex, part 2.

We will call this function hereafter the “modified output gap” (MOG). As opposed to the “classical output gap” (see Burda/Wyplosz (1997), p. 337), its economic intuition is related to the mismatch on the labour market: whenever the profile of labour demand and of labour supply differ significantly, an economic upswing (and a concomitant drop in the number of unemployed) will limit the rise in production to a level which is lower than the hypothetical output available when mismatch is absent. The reason lies in the inability of firms to fill the vacant positions on time. If the rise in production, hence, falls behind the rise in total demand, a higher inflation rate can be expected on the markets for goods and services. Leaving aside for a moment the direct costs of posting and “conserving” vacant positions in the firms, a significant vacancy ratio reflects specific higher costs: if there is an increase in total demand for goods, and the labour market cannot deliver enough additional qualified working force, three effects can be observed: On the one hand, firms will compete on scarce qualified labour force. This gives the latter a sort of “market power” and leads to higher wages. On the other hand, firms will invite their actual employees to work more hours. This is not for free. Thirdly, firms may try to increase the capital intensity of production. In doing so, they will often have to face higher interest rates. All of the three “channels” described lead to higher costs.

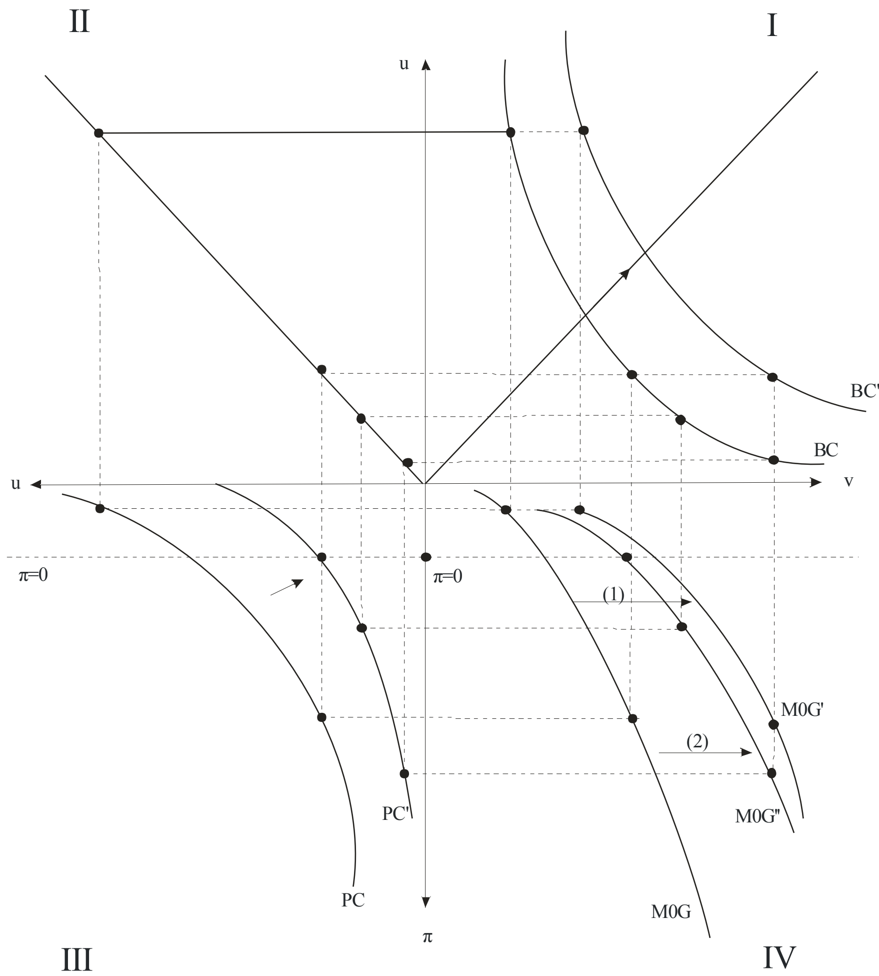
Is this a new insight? The concept presented here seems to be innovative for three reasons: While it is true that the Beveridge curve relates the vacancy ratio to the ratio of unemployment, whereas the Phillips curve, in turn, relates the rate of unemployment to the rate of inflation, simple economic logic suggests that, given these two pillars, another direct relationship between the vacancy ratio and the rate of inflation should exist. The same argument applies in principle to the widely respected AD curve (AD = “aggregate demand”), which in fact is won by simply combining the IS and the LM curve in the tradition of John Hicks and Alvin Hansen. Nobody, however, would put into question the merits of the AD curve and nobody is willing to renounce it within the discipline of macroeconomics. Secondly, the “modified output gap” (MOG) curve raises a point either totally neglected or, at best, treated only indirectly in the recent literature (see, e.g. Ravenna/Walsh (2007)[3]) - which points at the supply restraining and hence inflationary effect of the "mismatch" in the labour market. Thirdly, it is, contrary to the derivation of the AD curve, not trivial to identify the "correct" MOG curve.

In the following, we present a graphical analysis of the model. The first quadrant of Figure 1 depicts a convex Beveridge curve (BC) as equations (7) and (8) postulate. This curve will be shifted outwards (to BC´) if either the probability to be fired s increases and/or if the likelihood for getting a job η_0 drops, both effects tend to lower the efficiency of mediation on the labour market, and, at the same time, increase the relevant values of u and v, ceteris paribus. Given certain values of the unemployment rate along the Beveridge curve in the first quadrant, the second quadrant contains a line from the origin with an angle of 45 degrees which enables us to translate the corresponding unemployment rates into the third quadrant. Here we find two convex Phillips curves (PC, PC´) based on either equation (22) or equation (23). Let PC be the original Phillips curve. If, for whatever reasons, there is a decline in inflation expectations, this curve will be shifted inwards to its new position PC´. Notice that the line of price stability is a parallel below the axis of u and v; this enables us to consider negative inflation rates as well. With the set of vacancy ratios, unemployment rates and inflation rates determined jointly by the Beveridge and the Phillips curve, the fourth quadrant shows as the implied combinations of vacancy ratios and inflation rates. This new function with a positive slope shall be labeled "modified output gap" (MOG). Given the original Phillips curve (PC), the modified output gap curve (MOG) will be shifted to the right (arrow 1) to its new position MOG´, whenever the Beveridge curve moves outwards from BC to BC´. If, alternatively, the original Beveridge curve (BC) keeps its position, but the original Phillips curve (PC) moves inwards, the modified output gap curve (MOG) will shift also (arrow 2) to the right, say to its new position MOG´´.

The diagrams depicted in Figure 1 are able to describe the macroeconomic impact of the world economic crisis: The collapse of total demand has increased the unemployment

[3] "If vacancies could be posted free of charge, firms only need to pay workers a wage equal to worker's outside opportunity" (2007, p. 8) ... "Thus the marginal cost faced by retail firms would remain constant, as would inflation." (ibid, p. 15). Obviously, this is not the case and firms have to face higher marginal costs alongside an increase of the number of vacant jobs. This in turn, will raise the supply price of goods, ceteris paribus, and hence push up the rate of inflation.

Figure 1: Interaction between the Beveridge and the Phillips curve



Source: Own compilation.

rate in the affected economies and, at the same time, has decreased the inflation rate and the vacancy ratio (which equals a move upwards on the respective Beveridge curve and a move downwards on the respective Phillips curve). In terms of the modified output gap curve, the overall crisis can be traced as a move along a MOG curve towards the origin of the fourth quadrant. So far, the theoretical model seems to depict accurately from first glance what has happened between September 2008 and winter of 2010. It remains to be shown in the empirical section of the paper, to what extent this "initial conjecture" is also confirmed when econometric tools are applied to a well defined set of European countries' empirical data.

3 Empirical Evidence

3.1 The Data Set

The empirical data set is derived from EUROSTAT statistics, including information about vacancy ratios, inflation rates and unemployment rates on a quarterly basis for eleven of the “EA 12” countries right before, during and immediately after the peak of the world economic crisis.[4]

Notwithstanding the scarce questions about the definition and about the usage of the vacancy rate in reference to the Beveridge curve, the composition of the vacancy rate is so far still a variable with an open definition. On the one hand, some authors define the vacancy rate as the number of unfilled jobs expressed as a proportion of the labour force, which includes unemployed individuals and occupied posts in the denominator. On the other hand, there are both, authors and data sources like the EUROSTAT database, who choose a definition with vacancies and occupied posts in the denominator:

$$\begin{aligned} v_1 &= \frac{vacancies}{unemployed + occupied\ posts} \\ v_2 &= \frac{vacancies}{vacancies + occupied\ posts} = \frac{1}{1 + \frac{occupied\ posts}{vacancies}} \end{aligned} \tag{24}$$

From an methodological point of view, only the latter expression v_2 seems - in our opinion - to be reasonable due to the fact of a not arising problem of simultaneousness when regressing the unemployment rate u on the vacancy rate v. Because of the composition of the vacancy rate with vacancies in the numerator and the denominator the EUROSTAT data seem to be a very good source for testing the correlation of the Beveridge curve.

While it is neither clear when the financial crisis exactly started nor when it ended or if it has ended at all, we have picked up three different time periods in order to test “old theory”. The first time span begins with the collapse of Lehman Brothers in August/September 2008 (we start observations in the second quarter of 2008) and ends

[4]In January 1999 eleven European Union Member States adopted the Euro as their common currency and formed the Euro Area (EA). These eleven countries are also defined as the “EA 11” countries. Greece followed in January 2001 and specified in this respect the “EA 12” countries. In the meantime, this area expanded through a series of enlargements to 17 countries, so far. The EA 12 countries focused here include: Austria, Belgium, Germany, Greece, Finland, France, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain.

with the fourth quarter of 2009 what gives us seven observations for each of the eleven countries. The second time span chosen starts with the sub-prime mortgage crisis/the burst of the housing bubble in early summer (second quarter) of 2007 (we begin with the second quarter of 2007) and ends with the last quarter of 2009. This second sample gives us 11 observations for each of the eleven countries. The third time span defines the longest time period, starting with the second quarter of 2007 and ending with the fourth quarter in 2010 (15 observations).

It is a well known fact: when it comes to test the Beveridge curve empirically, one has to face a limited reliability of data concerning the vacancy ratio. Beyond this problem we found in particular, that Spain's vacancy ratio – in spite or due to the fact of the severe crisis it has to afford – nearly shows no variability at all in the time spans considered. It is unclear to which factors one can attribute this fact: It could either be that it reflects peculiarities of the Spanish labour market itself (see, for example, Pedraza (2007)). This market is segmented into two totally different parts: one formal sector with permanent contracts and a considerable degree of job protection and a second almost informal sector with only temporary contracts and no job protection at all. The second sector functions as a flexible buffer or a sort of "reserve army" and implies less if any fluctuation in the number of vacant jobs (see Sell (2010)), as firms have very little incentives to post vacant positions under these circumstances. Another possibility could be assigned to a potential reaction of the Spanish labour market if there is a simultaneous downturn of vacancies and occupied posts. As shown in the definitions above, this would lead to a constant ratio of these two variables according to the EUROSTAT definition.[5] This fact could also explain the special case of Spain.

Summing up: Every empirical analysis has to decide on the usage of a specific definition of the vacancy ratio. We have shown above the existing alternatives. The first definition could have the advantage of providing a higher variability of the vacancy ratio, especially in times of economic downturns. However, as discussed above, this

[5]As an alternative, we also analyzed quarterly data from the National Statistics Institute of Spain and from the Spanish Ministry Of Labour and Social Security to test the relationship of the Beveridge curve. The definition of the vacancy rate is based here on v_1 and considers the unemployed in the denominator. We found no (significant) relationship for the first time span (p-value = 0.814 an $Prob > F = 0.8144$). When we considered the two other time spans with more observations, we achieved a significant relationship with a p-value of 0.006 respectively a p-value of 0.005. But due to our methodological reasoning, we cannot exclude that the estimator is inconsistent. Please see the statistical Appendix (Table 4) for detailed results.

first definition suffers from a severe methodological problem of simultaneousness. The second definition of the vacancy ratio has the disadvantage to provide possibly a lower variability, but this possible disadvantage is compensated by the fact that consistent regression estimates can be expected. So in the end, we see a trade-off when choosing between the two definitions of the vacancy ratio. Because we regard the mentioned methodological problem of simultaneousness as more relevant, we do favor the latter definition v_2 used by EUROSTAT for our regressions.
Altogether, we can base our empirical analysis of the relationship between the vacancy ratio and the unemployment rate on 50 observations for the small sample, on 77 observations for the intermediate and on 113 observations for the large sample. The analysis of the Phillips curve is based on 77 observations for the small, on 121 observations for the intermediate and on 165 observations for the large sample.

3.2 Methodological Approach and a Brief Review of the Literature

The CESifo DICE REPORT has dedicated a full special section of its volume to "Europe's Labour Market and its crisis". The articles cover experiences gained in Spain, France, Germany, Netherlands, Ireland and Italy. In this contribution, we do not inspect the labour market country by country, but rather prefer to focus on a cross-country perspective. In order to gain valid and interpretable empirical results for our quarterly dataset with eleven different countries (EA 12 countries excluding Spain), we utilized a fixed effects model which keeps the country specific impact constant. To get rid of the omitted variables problem caused by the differences in the environments and in other factors of the respective countries, in principle two possibilities arose: either a dummy variable regression or the alternative of demeaning.
We found it more favorable for us to use a dummy variable regression than the demeaning alternative because we could additively derive country specific dummies, which are additionally interpretable. So we made use of dummy variables for every country ($d1$ to $d11$) and for all data sets. Thereafter, we conducted simple OLS regressions for the three data samples in order to investigate whether the relationships of the Beveridge curve and of the Phillips curve could be observed during the financial turmoil and the world economic crisis respectively.
Due to our theoretical analysis, we have chosen a simple OLS regression. Following

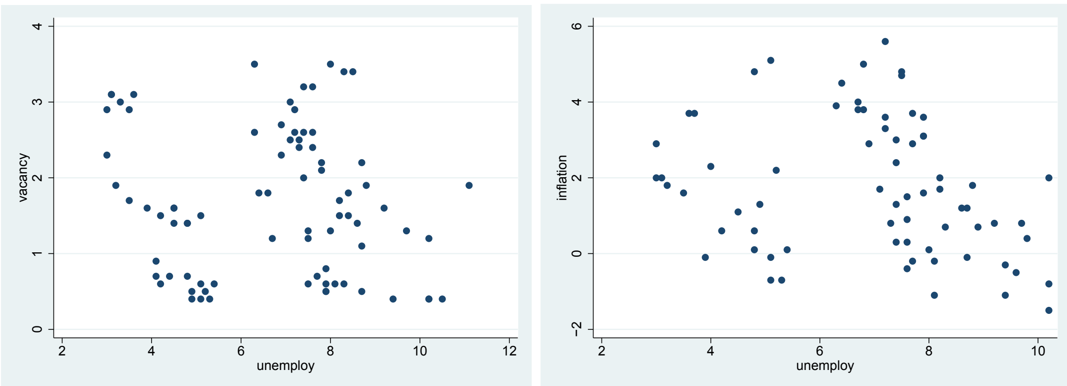
Blanchard/Diamond (1989) and Wall/Zoega (2002), we also logarithmized the endogenous variables and compared these estimations with our own results. In none of the cases, however, we found any contradictory evidence.
Furthermore, we were able to use the three data sets to analyze the relationship between the inflation rate and the vacancy ratio, which we have called above the "modified output gap" (MOG).
Different to the approach of testing a long-term relationship of central labour economic variables in one country (see for example Shiver (2005), who tests the Beveridge curve in the U.S. during the time period of 1951 and 2003), we want to investigate, whether the long-term conclusions also work out for two differentiations. Firstly, for a short-termed stress test, which is determined by an economic crunch of the recent financial crisis and secondly, not in a one country perspective but in a common sense of the Euro Area.
Other authors have already examined the possible existence of a "common" relationship (in the sense of the Beveridge curve, the Phillips curve and the modified output gap, MOG) making use of panel data. For instance, Boersch-Supan (1991) uses panel data of nine different regions in West Germany for the years from 1963 to 1986 to analyze the relationship of the Beveridge curve and finds only little support for a structural macroeconomic interpretation of the Beveridge curve in the data. DiNardo/Moore (1999), Turner/Seghezza (1999) and Furuoka/Munir (2009) focus on the relationship between the inflation rate and the rate of unemployment in the vein of the Phillips curve. Applying both an Ordinary Least Square regression (OLS) as well as General Least Square regression (GLS), DiNardo/Moore (1999) conclude for a set of nine different OECD countries that a strong and robust negative relationship existed between relative inflation and relative unemployment in the 1970s, 1980s, and 1990s. Turner/Seghezza (1999) confirm the results of DiNardo/Moore (1999) for 21 OECD countries during the time span between 1970s until 1997. The method chosen by these authors was the "Seemingly Unrelated Estimation Regression" (SURE). Furuoka/Munir (2009) also used a panel data set and applied OLS regression to analyze the possibly existing negative relationship between the unemployment rate and the inflation rate in five ASEAN countries during 1982-2004. However, their findings fall apart from the conclusions gained by DiNardo/Moore (1999) as well as by Turner/Seghezza (1999): They were unable to find a trade-off relationship for the countries within the time period considered.

Most recently, there are studies which directly test the New Keynesian Phillips curve (Scheufele (2010)) with the aim of identifying the frequency of price re-optimization by firms in the tradition of Calvo (1983), but also papers which test the trade-off relationship of the Phillips curve with observations since 2007 for a sample of industrialized countries: “Since the recession started in late 2007, evidence suggests that, consistent with the Phillips curve, a high level of unemployment has contributed to a decline in inflation.” (Liu/Rudebusch (2010), p. 1). Symmetrically, papers as the one by Kliesen (2010) not only confirm that the “Federal Reserve Bank of Philadelphia uses some variant of the Phillips curve to forecast inflation” (ibid., p. 7), but also anticipate the resurgence of higher employment along with more inflation in the next future.

3.3 Empirical Results

The three utilized data sets tell a straight forward story of the presumably “old” theories embedded in the Beveridge and in the Phillips curve. To start with, Figure 2 shows the scatter plots for vacancy ratios and unemployment rates (Beveridge curve) as well as for inflation rates and unemployment rates (Phillips curve) for the small sample (data from the second quarter of 2008 until the fourth quarter of 2009).[6]

Figure 2: Scatter plots for vacancy ratios and unemployment rates (lhs) for eight EA as well as for inflation rates and unemployment rates (rhs) for eleven EA during the financial crisis (7 observations each)



The scatter plot on the lhs includes data from Austria, Finland, Greece, Ireland, Luxembourg, Netherlands and Portugal, the one the rhs comprises all EA 12 countries excluding Spain. Source: Own figure.

[6]Scatter plots for a larger dataset can be found in the appendix (figure 3).

As the plots in Figure 2 insinuate, even (or in particular?) in times of a seemingly unpredictable, abnormal and extreme economic crisis traditional tools of labour economics and of macroeconomics are doing empirically fairly well. In order to yield a more well-founded statement, we were testing the relationships between inflation, unemployment and vacancy rates by a reduced-form estimation following Blanchard/Diamond (1989) as well as Wall/Zoega (2002) once again. But in our case we are focusing on the short-termed time span of the financial crisis.
In the first place, we had to derive the dummy variables for the different countries in order to get interpretable and valid empiric conclusions. Using the STATA package, we organized the dummies ($d1$ *to* $d10$) by alphabetical order. In each OLS regression we were able to leave one of the dummies out of the equation in order to achieve for every dummy the dependency from the omitted variable. In particular, we chose the dummy $d5$, which symbolizes Germany, to be left out of the regression. Hence, every dummy represents the country specific intercept vis-à-vis Germany which can be interpreted as the relative point of departure just before the downturn of the economies due to the worldwide financial crisis occurred.

The Beveridge curve

The pooled dummy variable OLS regression for the Beveridge curve makes use of observations for the vacancy ratio and for the unemployment rate. This applies for the small, the intermediate and the large sample. The estimated equation is of the following form:

$$\begin{aligned} vacancy_{it} = \beta_0 + \beta_1 unemploy_{it} + \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3 d_3 + \gamma_4 d_4 + \gamma_6 d_6 \\ + \gamma_7 d_7 + \gamma_8 d_8 + \gamma_9 d_9 + \gamma_{10} d_{10} + \gamma_{11} d_{11} + u_{it}. \end{aligned} \quad (25)$$

The results of the three OLS regressions we ran suggest a significant negative dependency of the vacancy ratio vis-à-vis to the unemployment rate. This applies to all data samples, as Table 1 reveals. The p-values of β_1 in the small sample denotes a significance level of ten percent whereas the intermediate and the big sample denote a significance level smaller than 5 percent. The observations for Belgium (just for the small sample), France and Italy were dropped out of the three regressions due to unavailability of continuous observations of the vacancy ratio during the time periods under consideration. Altogether, the explanatory power of the three regressions, with an $R^2 > 0.78$ is quite high.

Beyond this result, we find that all other dummies are significant and interpretable. In other words: In comparison to Germany, all other countries of the sample, such as Austria, Finland, Greece, Ireland, Luxembourg, Netherlands and Portugal have a smaller vacancy ratio in the initial situation. Such a result can have manifold reasons. It could be explained either by the fact of a better structural framework of the labour markets at each of those points in time (prominent skills shortage in Germany) or can perhaps be attributed to a different (and perhaps more accurate) data acquisition process for the vacancy ratio in Germany.

Table 1: Results of the OLS regressions for vacancy and unemployment rate – The Beveridge curve

Time	7 obs.		11 obs.		15 obs.	
Number of observations	50		77		113	
R-squared (R^2)	0.8425		0.8239		0.7836	
Adj. R-squared	0.8118		0.8002		0.7647	
Prob>F	0.000		0.000		0.000	
Root MSE	0.38553		.44842		.44716	
Ind. variable	**coef.**	**p-value**	**coef.**	**p-value**	**coef.**	**p-value**
Constant β_0	3.656	0.000	4.089	0.000	3.631	0.000
unemployment β_1	−.128	0.088	−.146	0.047	−.118	0.003
Dummy Austria γ_1	−1.571	0.000	−1.917	0.000	−1.403	0.000
Dummy Belgium γ_2	(dropped)		−.619	0.040	−.720	0.000
Dummy Finland γ_3	−1.022	0.000	−1.035	0.000	−.782	0.000
Dummy France γ_4	(dropped)		(dropped)		(dropped)	
Dummy Greece γ_6	−1.090	0.000	−1.335	0.000	−1.092	0.000
Dummy Ireland γ_7	−1.768	0.000	−1.985	0.000	−1.689	0.000
Dummy Italy γ_8	(dropped)		(dropped)		(dropped)	
Dummy Luxembourg γ_9	−2.532	0.000	−2.829	0.000	−2.502	0.000
Dummy Netherlands γ_{10}	−1.077	0.000	−1.129	0.004	−.977	0.000
Dummy Portugal γ_{11}	−1.924	0.000	−2.194	0.000	−1.950	0.000

Source: Own estimation.

The Phillips curve

The OLS regressions ran for the Phillips curve yielded a significant and negative relationship between the inflation rate and the unemployment rate for all data samples, as Table 2 demonstrates. The following equation was utilized for the estimation of the unknown parameters:

$$
\begin{aligned}
inflation_{it} = \beta_0 + \beta_1 unemploy_{it} + \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3 d_3 + \gamma_4 d_4 + \gamma_6 d_6 \\
+ \gamma_7 d_7 + \gamma_8 d_8 + \gamma_9 d_9 + \gamma_{10} d_1 0 + \gamma_{11} d_{11} + u_{it}.
\end{aligned} \tag{26}
$$

The conclusion from the following results seems clear: even during the hard times of a worldwide economic crisis triggered in turn by a financial market crisis, the "old" theory of the Phillips curve does pretty well when confronted with the empirical facts: All regressions support the convex relationship between the inflation rate and the unemployment rate with a p-value of zero. Though the measured R^2 is lower the

Table 2: Results of the OLS regression for inflation and unemployment rate - The Phillips Curve

Time	7 obs.		11 obs.		15 obs.	
Number of observations	77		121		165	
R-squared (R^2)	0.6441		0.5262		0.3879	
Adj. R-squared	0.5839		0.4784		0.3439	
Prob>F	0.000		0.000		0.000	
Root MSE	1.235		1.2023		1.3221	
Ind. variable	**coef.**	**p-value**	**coef.**	**p-value**	**coef.**	**p-value**
Constant β_0	11.099	0.000	9.233	0.000	5.462	0.000
unemployment β_1	−1.291	0.000	−.947	0.000	−.504	0.000
Dummy Austria γ_1	−3.863	0.000	−3.164	0.000	−1.376	0.011
Dummy Belgium γ_2	.632	0.342	−.083	0.872	.576	0.234
Dummy Finland γ_3	1.164	0.083	.012	0.981	.566	0.243
Dummy France γ_4	1.654	0.018	.493	0.341	.668	0.175
Dummy Greece γ_6	2.688	0.000	1.633	0.002	2.694	0.000
Dummy Ireland γ_7	1.722	0.019	−.498	0.333	−.183	0.713
Dummy Italy γ_8	.413	0.534	−.515	0.324	.235	0.627
Dummy Luxembourg γ_9	−2.767	0.000	−2.451	0.000	−.684	0.194
Dummy Netherlands γ_{10}	−5.147	0.000	−4.414	0.000	−2.185	0.000
Dummy Portugal γ_{11}	2.085	0.005	.949	0.077	.952	0.064

Source: Own estimation.

larger the sample, we still find a strong relationship between the dependent and the independent variables. Given the reference value for Germany (γ_5), the estimated dummy variables reveal over the three regressions that in the point of departure - before the economic crisis - Greece has a relatively higher rate of inflation whereas Austria and the Netherlands do show a relatively lower rate of inflation in their respective countries.

The modified output gap (MOG)

As one can see in the theoretical part, putting the two concepts of the Beveridge and the Phillips curve together has generated the explicit relationship between the inflation rate and the vacancy ratio which we have labeled the modified output gap (MOG). Given our previous theoretical reasoning about this relationship, the empirical correlation between these two macroeconomic variables should be positive. The idea in short was the following: The observed vacancy ratio can be understood as a signal for a significant mismatch or as a degree for skill shortage in the labour market. Is there a lack of skilled workers - respectively a rise of the vacancy ratio - wages will rise as well because of a higher bargaining power of the skilled workers. Employers will take this factor price effect into account and they will recalculate prices for goods, which will turn ceteris paribus to a higher observed inflation rate. The here applied equation for the OLS regression estimation to prove the supposed dependency reads as follows:

$$\begin{aligned} inflation_{it} = \beta_0 + \beta_1 vacancy_{it} + \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3 d_3 + \gamma_4 d_4 + \gamma_6 d_6 \\ + \gamma_7 d_7 + \gamma_8 d_8 + \gamma_9 d_9 + \gamma_{10} d_{10} + \gamma_{11} d_{11} + u_{it}. \end{aligned} \tag{27}$$

In spite of the smaller R^2 – implying a less good explanatory power of the regression equation in comparison to the regressions testing the relationships of the Beveridge and of the Phillips curve – all three estimated regressions indicate a positive and significant relationship between the inflation rate and the vacancy ratio. Again, because of the unavailability of some of the vacancy ratio data, information for France and Italy had to be dropped in all three samples. In addition, the data for Belgium had to be left out in the case of the small observation sample. Most dummy variables explain a higher inflation rate in the initial situation, and this effect was estimated to be significant for Finland, Greece, and Luxembourg. The only exception is Ireland, which shows in all three estimations a negative effect and in the longest time period even a significant negative effect.

Table 3: Results of the OLS regression for inflation and vacancy rate - The modified output gap

Time	7 obs.		11 obs.		15 obs.	
Number of observations	50		77		113	
R-squared (R^2)	0.4346		0.4063		0.4807	
Adj. R-squared	0.3243		0.3265		0.4353	
Prob>F	0.0016		0.000		0.000	
Root MSE	1.5715		1.4036		1.2785	
Ind. variable	**coef.**	**p-value**	**coef.**	**p-value**	**coef.**	**p-value**
Constant β_0	1.991	0.262	−.965	0.412	−.034	0.966
vacancy β_1	1.221	0.054	.926	0.015	.597	0.029
Dummy Austria γ_1	.617	0.620	.024	0.981	.090	0.886
Dummy Belgium γ_2	(dropped)		.575	0.541	.978	0.117
Dummy Finland γ_3	2.606	0.016	1.568	0.026	1.114	0.031
Dummy France γ_4	(dropped)		(dropped)		(dropped)	
Dummy Greece γ_6	2.785	0.018	2.404	0.004	2.531	0.000
Dummy Ireland γ_7	−.140	0.936	−1.058	0.402	−1.839	0.030
Dummy Italy γ_8	(dropped)		(dropped)		(dropped)	
Dummy Luxembourg γ_9	3.145	0.056	2.706	0.013	2.093	0.006
Dummy Netherlands γ_{10}	.920	0.315	.283	0.651	.129	0.791
Dummy Portugal γ_{11}	1.985	0.219	1.785	0.102	1.074	0.162

Source: Own estimation.

4 Conclusions

How do traditional concepts of labour economics and of macroeconomics perform empirically in light of the financial and the overall economic crisis? To address this question we have proceeded as follows: In the first part of this paper, we have derived the traditional Beveridge curve. Then we have modified the New Keynesian Phillips curve so that the traditional "trade off" between the unemployment rate and the inflation rate becomes again transparent. In the second stage of the paper, we have combined the Beveridge and the New Keynesian Phillips curve to create a so-called "modified output gap"(MOG). This economic relationship between the rate of inflation and the vacancy ratio demonstrates that the inability to find appropriate jobs (the labourers' view) or to fill positions with adequate candidates (the entrepreneurs' view) tends to put an upward pressure on the inflation rate: be it because there is no expansion in the supply of goods and services high enough to match the rise in total demand or be it because such an expansion seems only viable for the entrepreneurs if they pay premiums on wages to compensate for the extra time work and hence charge higher

prices.
The third section of the paper tests the three mentioned functions for a number of well defined European countries and the period of time between the outbreak of the financial and economic crisis and the preliminary end of the most acute phase of the crisis. The results show that all three relationships perform surprisingly well empirically and it seems that they do well not despite, but perhaps just because of the depth and size of the overall economic crisis. One should, therefore, be quite careful when it comes to throw (more or less) old economic theories into the tons of garbage of the history of economic thought.

5 Policy Implications

As our paper shows, the old controversy between "supply-side economics" and "demand-side economics" is obsolete when we come to the field of economic policy: on the one hand, demand management has to take into account the supply restraining (enhancing) and hence inflationary (deflationary) effect of the (reduction of) mismatch in the labour market. Policies oriented towards the supply side of the economy should, on the other hand, carefully register the possible moves in the expectations of inflation (deflation).
Combining smartly demand and supply side policies will hence, increase effectiveness and efficiency of both of them. Each of these policies alone, can easily fail: improving the mediation in the labour market when a lack in total demand is the key reason for a crisis, can hardly overcome unemployment. As far as demand policies are concerned, a second "policy ineffectiveness lemma" emerges: at high levels of mismatch, expansive measures of monetary and/or fiscal policies will lead to higher inflation in the first place with little (positive) repercussions on the real side of the economy.

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6 Appendix

6.1 Mathematical Annex Part 1

Few simple algebraic manipulations of (5) gives us equations (28), (29) and (30).

$$\frac{sA}{\eta_0} = (V \cdot U)^{0,5} \tag{28}$$

$$\left(\frac{s}{\eta_0}\right)^2 \cdot A^2 = V \cdot U \tag{29}$$

$$U = \left(\frac{s}{\eta_0}\right)^2 \frac{A^2}{V}. \tag{30}$$

Next, we introduce the ratio of vacancies $v(v = V/\bar{A}^A)$, where $\bar{A}^A$ can be taken as the time invariant supply of labour, into (30) which gives us:

$$U = \left(\frac{s}{\eta_0}\right)^2 \cdot \frac{A^2}{v \cdot \bar{A}^A} \tag{31}$$

or respectively

$$\frac{U \cdot \bar{A}^A}{A^2} = \left(\frac{s}{\eta_0}\right)^2 \cdot \frac{1}{v}. \tag{32}$$

Analogously, we define the unemployment ratio $u(u = U/\bar{A}^A \quad or \quad U = u \cdot \bar{A}^A)$. This definition enables us to further modificate our equation, taking into account that $A/\bar{A}^A \equiv 1 - u$:

$$\bar{A}^A \frac{U}{A^2} = \frac{u \cdot \bar{A}^{A^2}}{(1-u)^2 \cdot \bar{A}^{A^2}} = \frac{u}{1-u^2}. \tag{33}$$

Plugging (33) into (32) gives finally:

$$\frac{U \cdot \bar{A}^A}{A^2} = \frac{u}{(1-u)^2} = \left(\frac{s}{\eta_0}\right)^2 \cdot \frac{1}{v}. \tag{34}$$

In the following, we will solve equation (34) for v:

$$\frac{v \cdot u}{(1-u)^2} = \left(\frac{s}{\eta_0}\right)^2 \tag{35}$$

$$v = \left(\frac{s}{\eta_0}\right)^2 \cdot \frac{(1-u)^2}{u} = \left(\frac{s}{\eta_0}\right)^2 \cdot \frac{1-2u+u^2}{u} \tag{36}$$

$$v = \left(\frac{s}{\eta_0}\right)^2 \cdot \left[\frac{1}{u} - 2 + u\right]. \tag{37}$$

6.2 Mathematical Annex Part 2

Rearranging the Beveridge curve:

$$v = \left(\frac{s}{\eta_0}\right)^2 \cdot \frac{(1-u)^2}{u} \tag{38}$$

gives us an expression which is in fact a quadratic term. Such a term usually has two solutions:

$$uv = \left(\frac{s}{\eta_0}\right)^2 \cdot (1-u)^2 \tag{39}$$

$$uv = \left(\frac{s}{\eta_0}\right)^2 \cdot (1-2u+u^2) \tag{40}$$

$$uv = \left(\frac{s}{\eta_0}\right)^2 - 2u\left(\frac{s}{\eta_0}\right)^2 + \left(\frac{s}{\eta_0}\right)^2 u^2 \tag{41}$$

$$0 = \left(\frac{s}{\eta_0}\right)^2 - 2u\left(\frac{s}{\eta_0}\right)^2 + \left(\frac{s}{\eta_0}\right)^2 u^2 - uv \tag{42}$$

$$\left(\frac{s}{\eta_0}\right)^2 u^2 - \left(2\left(\frac{s}{\eta_0}\right)^2 + v\right) u + \left(\frac{s}{\eta_0}\right)^2 = 0 \tag{43}$$

$$u_{1,2} = \frac{2\left(\frac{s}{\eta_0}\right)^2 + v \pm \sqrt{(-(2(\frac{s}{\eta_0})^2+v))^2 - 4(\frac{s}{\eta_0})^4}}{2(\frac{s}{\eta_0})^2} \tag{44}$$

$$u_{1,2} = \frac{2\left(\frac{s}{\eta_0}\right)^2 + v \pm \sqrt{4(\frac{s}{\eta_0})^4 + 4(\frac{s}{\eta_0})^2 v + v^2 - 4\left(\frac{s}{\eta_0}\right)^4}}{2(\frac{s}{\eta_0})^2} \tag{45}$$

$$u_{1,2} = \frac{2\left(\frac{s}{\eta_0}\right)^2 + v \pm \sqrt{4(\frac{s}{\eta_0})^2 v + v^2}}{2(\frac{s}{\eta_0})^2} \tag{46}$$

$$u_1 = 1 + \frac{v}{2\left(\frac{s}{\eta_0}\right)^2} + \frac{\sqrt{4\left(\frac{s}{\eta_0}\right)^2 v + v^2}}{2\left(\frac{s}{\eta_0}\right)} \tag{47}$$

$$\frac{\partial u_1}{\partial \upsilon} = \frac{1}{2\left(\frac{s}{\eta_0}\right)^2} + \frac{4\left(\frac{s}{\eta_0}\right)^2 + 2\upsilon}{4\left(\frac{s}{\eta_0}\right)^2\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}} = \frac{1}{2\left(\frac{s}{\eta_0}\right)^2} + \frac{2\left(\frac{s}{\eta_0}\right)^2 + \upsilon}{2\left(\frac{s}{\eta_0}\right)^2\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}} > 0 \quad (48)$$

This first solution is mathematically correct, but it lacks economic meaning because $u_1 = 1 + ... > 1$ is beyond the range of permitted values for the unemployment rate. The second solution reads:

$$u_2 = 1 + \frac{\upsilon}{2\left(\frac{s}{\eta_0}\right)^2} - \frac{\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}}{2\left(\frac{s}{\eta_0}\right)^2} \quad (49)$$

$$\frac{\partial u_2}{\partial \upsilon} = \frac{1}{2\left(\frac{s}{\eta_0}\right)^2} - \frac{4\left(\frac{s}{\eta_0}\right)^2 + 2\upsilon}{4\left(\frac{s}{\eta_0}\right)^2\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}} = \frac{1}{2\left(\frac{s}{\eta_0}\right)^2} - \frac{2\left(\frac{s}{\eta_0}\right)^2 + \upsilon}{2\left(\frac{s}{\eta_0}\right)^2\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}} < 0 \quad (50)$$

This solution is both mathematically correct and economically meaningful, now that the unemployment rate is in the range of plausible values, situated between 0 and 1. The explicit form of the MOG curve can now be gained by introducing the solution u_2 into the Phillips curve as given by equation (31):

$$\pi_t = \beta\pi_t^e + \kappa\gamma\left[\tilde{u}_t - \left(1 + \frac{\upsilon}{2\left(\frac{s}{\eta_0}\right)^2} - \frac{\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}}{2\left(\frac{s}{\eta_0}\right)^2}\right)\right] + \varepsilon_t \quad (51)$$

$$\pi_t = \beta\pi_t^e + \kappa\gamma\tilde{u}_t - \kappa\gamma - \frac{\kappa\gamma\upsilon}{2\left(\frac{s}{\eta_0}\right)^2} + \frac{\kappa\gamma\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}}{2\left(\frac{s}{\eta_0}\right)^2} + \varepsilon_t \quad (52)$$

$$\frac{\partial \pi_t}{\partial \upsilon} = -\frac{\kappa\gamma}{2\left(\frac{s}{\eta_0}\right)^2} + \frac{\kappa\gamma\left(4\left(\frac{s}{\eta_0}\right)^2 + 2\upsilon\right)}{4\left(\frac{s}{\eta_0}\right)^2\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}} > 0 \quad (53)$$

Alternatively, we can introduce the solution u_2 into the Phillips curve as given by equation (32):

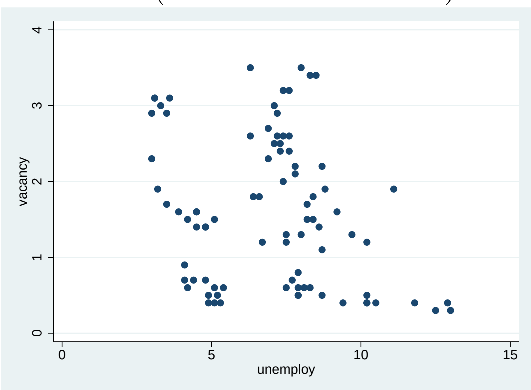
$$\pi_t = (1-\varphi)\pi_t^e + \varphi\pi_{t-1} + \kappa\gamma\tilde{u}_t - \kappa\gamma\left(1 + \frac{\upsilon}{2\left(\frac{s}{\eta_0}\right)^2} - \frac{\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}}{2\left(\frac{s}{\eta_0}\right)^2}\right) + \varepsilon_t \quad (54)$$

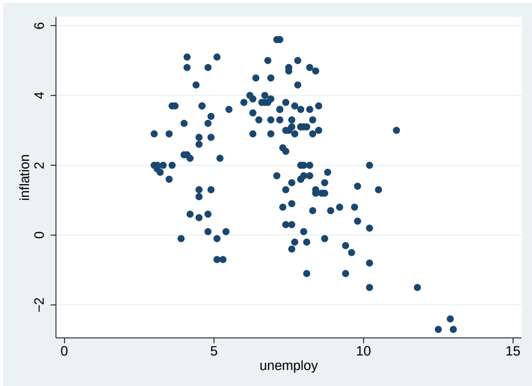
$$\pi_t = (1-\varphi)\pi_t^e + \varphi\pi_{t-1} + \kappa\gamma\tilde{u}_t - \kappa\gamma - \frac{\kappa\gamma\upsilon}{2\left(\frac{s}{\eta_0}\right)^2} + \frac{\kappa\gamma\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}}{2\left(\frac{s}{\eta_0}\right)^2} + \varepsilon_t \tag{55}$$

$$\frac{\partial\pi_t}{\partial\upsilon} = -\frac{\kappa\gamma}{2\left(\frac{s}{\eta_0}\right)^2} + \frac{\kappa\gamma\left(4\left(\frac{s}{\eta_0}\right)^2 + 2\upsilon\right)}{4\left(\frac{s}{\eta_0}\right)^2\sqrt{4\left(\frac{s}{\eta_0}\right)^2\upsilon + \upsilon^2}} > 0 \tag{56}$$

6.3 Statistical Appendix

Figure 3: Scatter plots for vacancy ratios and unemployment rates (lhs) for eight EA as well as for inflation rates and unemployment rates (rhs) for eleven EA during the financial crisis (12 observations each)





The scatter plot on the lhs includes data from Austria, Finland, Greece, Ireland, Luxembourg, Netherlands and Portugal, the one the rhs comprises all EA 12 countries excluding Spain. Source: Own figure.

Table 4: Results of the OLS regression for vacancy and unemployment rate in Spain – The Spanish Beveridge curve

Time	7 obs.		11 obs.		15 obs.	
Number of observations	7		11		15	
R-squared (R^2)	0.0121		0.5873		0.4614	
Adj. R-squared	-0.1855		0.5414		0.4200	
Prob>F	0.8144		0.006		0.0054	
Root MSE	0.2699		.4505		.47863	
Ind. variable	**coef.**	**p-value**	**coef.**	**p-value**	**coef.**	**p-value**
Constant β_0	3.535	0.001	5.362	0.000	5.050	0.000
unemployment β_1	$-.008$	0.814	$-.116$	0.006	$-.086$	0.005

Source: Own estimation.

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