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# Regional Income Distribution and Human Capital Formation: A Model of Intergenerational Education Transfer in a Global Context

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# Regional Income Distribution and Human Capital Formation*

A Model of Intergenerational Education Transfer in a Global Context

Florian W. Bartholomae, Alina M. Popescu

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## Abstract

The demographic problems in developed countries are getting more and more important. Very low fertility rates especially among skilled individuals will soon become relevant for a country's economy. Also of importance is education of children. Since there is an increasing demand for skilled workers, the positive correlation between social background and education worsens the situation. Therefore family planning as well as fertility providing and educational measures are of major importance for regional decision makers.

We define in our model the optimal number of children considering the income and education of their parents by using a Cobb–Douglas utility function which implies that children and consumption are complementary goods. Children are considered to be a differentiated good with respect to their education. Therefore, we distinguish between high educated and low educated children. After deciding the optimal number of children, the education level of children has to be determined. We assume that only one parent is responsible for the education. Further we presume a negative correlation between the opportunity costs of educating a child and their parent's qualification. Since we consider the parents income and education, many cases result.

Regional policy makers have the possibility to change individual decisions regarding offspring by creating monetary incentives. As wages and therefore family income are exogenous, the regional governments have only two policy measures left: either child allowance and/or scholarships. Considering the population's preferences, regions may optimize the number and structure of children.



Keywords: population policy, education, qualification, factor proportions, globalization

JEL–classification: D31, D33, J12, J31, R23

# 1 Introduction

Demographic questions like fertility issues, aging as well as the consequences of demographic changes on economy have been in the last decades an important part of economic research. This paper deals with the impact of parents' education on the quantity and quality of children. Since there are theoretical and empirical evidences regarding the negative correlation between quantity and quality of children (Becker and Lewis, 1973) and the negative correlation between parents education and quantity of children (Michael, 1973) we analyze in a static theoretical framework the parents behavior and decision making regarding their offspring. We therefore assume that only one parent is responsible for educating the children. The education decision is taken with respect to child preferences, opportunity costs and parents education. Many cases arise. Two argumentative trends are important for this analysis:

First, the desired family size has to be determined. Many aspects have to be considered. For analyzing the family fertility decision in an economic way, a definition of "child" is necessary. In the literature three main definitions can be found.

- Children can be seen as a zero–utility by–product of sexual activity (Easterlin, 1969; Cochrane, 1975) where the optimal family size is determined by a combination of duration of marriage and standard of living and a subsistence standard of living which cannot be undercut (Cochrane, 1975; Eversley, 1959; Schultz, 1973).

- Furthermore, children can be interpreted as being investment goods (De Tray, 1973). In this case there is a correlation between educating children (investing in children) and a future income. Children are home–produced durable assets which allow parents to consume services whereas these services depend on the biological units of children (quantity) and the resource intensity which is invested in the children's education (quality) (De Tray, 1973). Optimal family size is resulting from the inter–temporal consumption decision. Since many social security services assume continuous demographic development, this definition is very familiar.

- The third way to handle with the fertility decision is by assuming children as consumption good. In this case a trade–off between children and consumption of other goods, whereas children are normal goods (Okun, 1959), arise.

In our analysis we use the third definition.

Second, the decision regarding quality of children has to be taken. Therefore we consider the parents education and assume more family types. Opportunity costs (time costs) as well as prices of children are important variables for taking into account, since there is an effect on quantity and quality of children. The prices of children are increasing with increasing quantity (the costs for an additional child and constant quality) and with increasing quality (the cost for an additional unit quality and constant quantity) (Becker and Lewis, 1973). The negative correlation between the parents' education and the quantity of children may be explained through the fact that increases in the time value of the educating parent is increasing the prices for children and is lowering the fertility (Michael, 1973).

Since we consider children as being a consumption good, and not an investment good we do not examine that children expenditures are positively correlated with parent's future utility (Becker, 1960). We consider in our paper for the analysis of the quality of children not only the correlation between quantity and quality and the relationship between the parent's education and the quantity of children but as well the connection between parent's education and quality of children. We determine under which circumstances educated (skilled) and non–educated (unskilled) parents have educated children.

The quality dimension of children is considered in this paper as being important for a region. High educated children are an important location factor and therefore utility maximizing for regions.

After identifying the quantity and quality of children in section 2, we analyze this decision in a regional context in section 3. Regions, defined in our analysis as small countries in a global context, maximize their utility with respect to their factor endowment (skilled and unskilled labor) considering the wages as exogenous. Assuming a standard 2x2x2 Heckscher–Ohlin Model, a GDP maximizing quantity and quality of children results for the region. Finally section 4 concludes.

## 2 The basic model

In our model we analyze first the family decision regarding offspring considering the prevailing preferences and prices in the region. We consider on the one side the family utility function and therefore the educational level of the parents. The educational level has been taken into consideration since there is a lot of theoretical and empirical evidence that shows the negative correlation between education and fertility (Michael, 1973; Cochrane, 1975; Becker, Murphy and Tamura, 1990; Morand, 1999; Toor, 2007). Furthermore of importance is the impact of the parent's social background on the children's education.

We assume a trade–off between consumption and offspring. Utility–maximizing households decide about that amount of their income that they are disposed to spend for offspring. Since we presume a Cobb–Douglas utility function, consumption as well children are necessary to generate a positive utility (e. g. if the household decides to have no children, then its utility will be zero). Hence following maximizing problem arises:[1]

$$U = (n_N + An_E)^{\tau}\, C^{1-\tau} \quad \text{s. t.} \quad p^i_N n_N + p^i_E n_E + p_C C = y, \tag{1}$$

[1]Of course we are aware of the characteristic of the utilization of children in the utility function as being a normal consumption good, whose demand is strictly increasing in income and decreasing in the price. This contradicts the empirical findings, that demand for children decreases with income. However, we consider only a static framework, where prices as well as income do not change. In a dynamic framework prices of children would rise with increasing income, since richer families require a higher standard of living of their children.

where $n_N$ is the number on non–educated children, $n_E$ is the number of educated children, $\tau$ is share of expenditures for children and hence $1 - \tau$ is the share spend on consumption $C$. The budget constraint gives us the proportioning of consumption and offspring considering the family income $y$ and all prices (i. e. for educated children, non–educated children and consumption). We consider the price of consumption as numeraire. Prices of children depend on several factors like general cost of living, child–care and tuition fees or additional costs due to a need of more living space. Hence, if we compare several regions, the maximization problems differ.

Furthermore we assume the offspring to be a differentiated good with respect to their education. There are high–educated children and less–educated children. The household's preferences regarding the children's education is given in the utility function by the marginal rate of substitution $A$. Since the income of households depends on the educational level of its adult members and education involves costs, the education decision for the offspring will be taken after considering the costs for education.

We consider many types of families by differentiating the educational level of the parents. A family may consist of two skilled members, two unskilled members or a skilled and an unskilled member. The interesting results can occur in the latter case, since the optimal number and later education–level of children is not obvious.

Given the utility function and the budget constraint, the relative prices and hence the relative costs for children considering the education level of the parents gives us the condition whether to invest in education of children or not. If $p_E/p_N < A$, a family decides only for educated children whereas in the case of $p_E/p_N > A$, parents will only have uneducated children. Furthermore the following condition holds

$$\left(\frac{p_E}{p_N}\right)_{L_S} < \left(\frac{p_E}{p_N}\right)_{L_U}, \tag{2}$$

where $L_S$ is the skilled parent and $L_U$ the unskilled one responsible for the education of the children, assuming that only one parent is responsible for the education of the family offspring.

The decision regarding the education responsibility is taken by considering the opportunity costs—time–costs that are necessary for the children's education that leads to a loss of family income—and the marginal rate of substitution $A$—preference parameter which gives us the bias for educated children. The time–costs for an uneducated child are for all types of households the same. The time–costs for educating a child depends on the educational level of the parents. We assume that the higher the parent's educational level the lower the time–cost for educating a child. This assumption is supported by the fact that high–skilled parents own the necessary knowledge for educating and supporting their children in a shorter time than unskilled parents (Michael, 1973). This implies that a skilled parent spends $\varphi_N$ for educating a child ($\varphi_N$ are that time–costs which are spend by an unskilled parent for an uneducated child, $\varphi_E$ are the time–costs to educate children).

$$\varphi_N(\theta_i) = \varphi_N \quad \text{and} \quad \varphi_E(\theta_i) = \varphi_N/\theta_i \tag{3}$$

where $\theta_S$ and $\theta_U$ denotes the parent–qualification parameter of the skilled respectively the unskilled family member. It is defined as being in the interval $]0,1)$. It measures the possibilities for each parent to educate its child. I. e. the higher $\theta$ the higher the parent's education and the more easily a better education of the children. We set $\theta_S = 1 > \theta_U$. Hence we get $\varphi_E(\theta_S) = \varphi_N$ and $\varphi_E(\theta_U) = \varphi_N/\theta_U$. Iff the parent responsible for child–care is unskilled then $\varphi_E > \varphi_N$ and iff the parent is skilled then $\varphi_E = \varphi_N$. $\varphi_N$, $\theta_U$ and hence $\varphi_E$ may differ across regions due to different supplies of childcare like kindergartens, all–day schools or youth centers.

Since the opportunity costs are higher for high qualified parents (the income loss caused by the time needed for the education of the children) a situation where skilled parents have few educated children and unskilled parents have many unskilled children may occur. In families consisting of a skilled and an unskilled parent, the number and educational level of children is decided considering the family preferences and the relative wages which will be determined in a general equilibrium setting in the later analysis.

The maximization of the utility function as defined in (1) gives the following results,

$$n^*_{E+N} = \tau \frac{y}{p_{E+N}} \quad \text{and} \quad C^* = (1-\tau)y, \tag{4}$$

where $n^*_{E+N}$ is the (optimal) number of children (educated as well as non–educated), $y$ is the total income of the family, $p_{E+N}$ the "price" of children, $\tau$ is the income share spend on children and $C^*$ are the (optimal) consumption expenditures. To determine the optimal number of non–educated and educated children from the total number, we derive two cases depending on the relative price of education compared to the preference (bias) for education. These two cases are

Case (i) $p_E/p_N > A$: $n^*_N = \tau \frac{y}{p_N}$, $n^*_E = 0$ and $C^* = (1-\tau)y$.

Case (ii) $p_E/p_N < A$: $n^*_N = 0$, $n^*_E = \tau \frac{y}{p_E}$ and $C^* = (1-\tau)y$.

Since we measure opportunity costs in income losses which arises from less time available for working, we have to consider time–costs in our budget constraint in (1). We get

$$\left(p_N + \varphi_N w^{(1)} + \varphi_N w^{(2)}\right) n_N + \left(p_E + \varphi_E w^{(1)} + \varphi_E w^{(2)}\right) n_E + C - w^{(1)} - w^{(2)}, \tag{5}$$

where the first bracket term are the opportunity costs for non–educated children and the second bracket term are the opportunity costs for educated children. We consider the income of both parents (noted with $w^{(1)}$ and $w^{(2)}$) in the calculation of the opportunity costs since the education decision within the family did not take place yet. In the latter analysis after the education decision $w^{(1)}$ or $w^{(2)}$ will equal zero reducing the opportunity costs, since only one parent is taking care of the children's education. For calculating the optimal number and education–type of children for each case we need the relative opportunity costs for educated children that are given by:[2]

$$\frac{p_E + \varphi_E w^{(1)} + \varphi_E w^{(2)}}{p_N + \varphi_N \left(w^{(1)} + w^{(2)}\right)} \tag{6}$$

By inserting (6) in the cases (i) and (ii)—with respect to the relation between the opportunity costs and the preference parameter $A$—and considering first the family types that consist of equal–skilled members, we obtain the following results for the optimal number of children:

| | $L_S L_S$ | $L_U L_U$ |
|---|---|---|
| $p_E/p_N > A$ | $\frac{2\tau w_S}{p_N+\varphi_N w_S(1+\tau)}$ | $\frac{2\tau w_U}{p_N+\varphi_N w_U(1+\tau)}$ |
| $p_E/p_N < A$ | $\frac{2\tau w_S}{p_E+\varphi_N w_S(1+\tau)}$ | $\frac{2\tau w_U}{p_E+\varphi_N w_U(1+\tau)/\theta_U}$ |

Resume that $p_E/p_N > A$ implies that only non–educated children will be brought up. Since $p_E > p_N$ and $\varphi_N < \varphi_N/\theta_U$ both family types will have more non–educated than educated children. On the one hand it is more (time–)cost intensive for unskilled families to educate children and on the other hand, these families are poorer, a fact that further lowers the optimal number of educated children further. Therefore it is efficient that only the skilled families educate their offspring.[3] If their wages are higher than $w_U$ the positive income effect increases the number of educated children.

Since the number of educated children depends on the preference parameter $A$ as well as on the relative price for educated children, we derive the necessary condition to ensure that skilled parents have educated children and unskilled parents have non–educated offspring. Considering (2) $A$ is hence assumed to be as follows:

$$\frac{p_E + \varphi_N w_S}{p_N + \varphi_N w_S} < A < \frac{p_E + \frac{\varphi_N}{\theta_U} w_U}{p_N + \varphi_N w_U}, \tag{7}$$

[2]Note that $\varphi_E$ is different for different skilled parent, cf. (3), whereas $\varphi_N$ is the same.
[3]Skilled families have a kind of comparative advantage because of a higher productivity in child education.

since otherwise only offspring of one education level results. The first inequation gives the condition $p_E + \varphi_N w_S < A p_N + A \varphi_N w_S$ which simplifies to $p_E < \varphi_N w_S (A-1) + A p_N$. The second inequation shows that $p_E > (A - 1/\theta_U)\, \varphi_N w_U + A p_N$. Both inequalities are fulfilled iff

$$\frac{w_S}{w_U} > \frac{A - 1/\theta_U}{A - 1}. \tag{8}$$

Consider this inequality as **condition (1)**. Iff condition (1) is fulfilled, then skilled parents have **only educated** children. Analogous unskilled parents have **only unschooled** children.

An interesting question arising from this consideration is, under which conditions unskilled parents have more children than skilled parents. Considering that the education of children depends on the education of the parents (cf. condition (1)), following inequation arises:

$$\frac{2\tau w_U}{p_N + \varphi_N w_U (1+\tau)} > \frac{2\tau w_S}{p_E + \varphi_N w_S (1+\tau)} \tag{9}$$

This simplifies to

$$\frac{w_S}{w_U} < \frac{p_E}{p_N}, \tag{10}$$

which is **condition (2)**. Iff condition (2) is fulfilled, then skilled parents have **less** children and unskilled parents **more** children.

These two conditions allow us to state the following proposition:

**Proposition 2.1** *Skilled parents ($L_S L_S$) have higher educated but less children whereas unskilled parents ($L_U L_U$) have less educated and more children, iff the prevailing wage ratio in the considered region satisfies* $(A - 1/\theta_U)/(A-1) < w_S/w_U < p_E/p_N$.

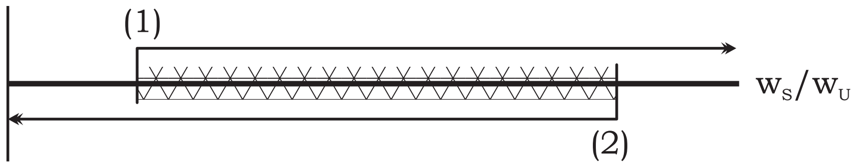
**Proof** We have to show that the set described in the proposition is not an empty set. Iff condition (1) and (2) holds, then

$$\frac{p_E}{p_N} > \frac{A - 1/\theta_U}{A - 1}$$

have to be truth. Since $p_E/p_N > 1$ and $(A - 1/\theta_U)/(A-1) \leq 1$ due to $\theta_U \in [0,1[$ this inequation alway holds. ∎

Figure 1 illustrates proposition 2.1. The lower $(A - 1/\theta_U)/(A-1)$ (the necessary value, that skilled parents have educated offspring) the more separation is ensured. This is given the lower the preference for educated children (low values of $A$), and the more unproductive unskilled parents in the education of their children (low values of $\theta_U$). For a large set of values, this condition becomes negative implying that for every relative wage skilled parents will have educated children.[4] On the other hand, the higher are costs for education, $p_E$, compared to the price for children, $p_N$, the less children skilled parents have This expands the set where proposition 2.1 is valid.

Figure 1: Graphical illustration of proposition 2.1




Source: Own model as described in text.

We now consider the results of families consisting of differently skilled members.

| | $L'_S L_U$ | $L_S L'_U$ |
|---|---|---|
| $p_E/p_N > A$ | $\frac{\tau(w_S+w_U)}{p_N+\varphi_N w_S(1+\tau)}$ | $\frac{\tau(w_S+w_U)}{p_N+\varphi_N w_U(1+\tau)}$ |
| $p_E/p_N < A$ | $\frac{\tau(w_S+w_U)}{p_E+\varphi_N w_S(1+\tau)}$ | $\frac{\tau(w_S+w_U)}{p_E+\varphi_N w_U(1+\tau)/\theta_U}$ |

[4]The necessary condition is $A < 1/\theta_U$.

Resume our assumption, that only one family member is responsible for child–care. We therefore distinguished between the responsibility for this duty. In the table the family member responsible for child–education is marked with an apostrophe. We assume further that condition (1) holds, too. Please note that we do not consider the intern family decision process regarding the responsible educating parent. This may arise from individual preferences not captured in our utility function. Again we see that if a family decides to have educated children, their number will be less than that of non–educated offspring.

According to the derivation of condition (2) we get

$$\frac{\tau(w_S + w_U)}{p_N + \varphi_N w_U(1+\tau)} > \frac{\tau(w_S + w_U)}{p_E + \varphi_N w_S(1+\tau)}. \tag{11}$$

This simplifies to

$$w_S > \frac{p_N - p_E}{\varphi_N(1+\tau)} + w_U \tag{12}$$

which is **condition (3)**. Iff condition (3) is fulfilled, then a skilled parent has **less** children and an unskilled parent has **more** children.

To sum up, in the general case if we consider all types of families, we have the following conditions:

**Condition (1)** $w_S > \frac{A-1/\theta_U}{A-1} w_U$

**Condition (2)** $w_S < \frac{p_E}{p_N} w_U$

**Condition (3)** $w_S > \frac{p_N - p_E}{\varphi_N(1+\tau)} + w_U$

These allow us to state the following proposition:

**Proposition 2.2** *Whenever a skilled parent is responsible for child education, their children will be more educated but their number will be less than if an unskilled parent is responsible, iff the wage gap in this region is close.*

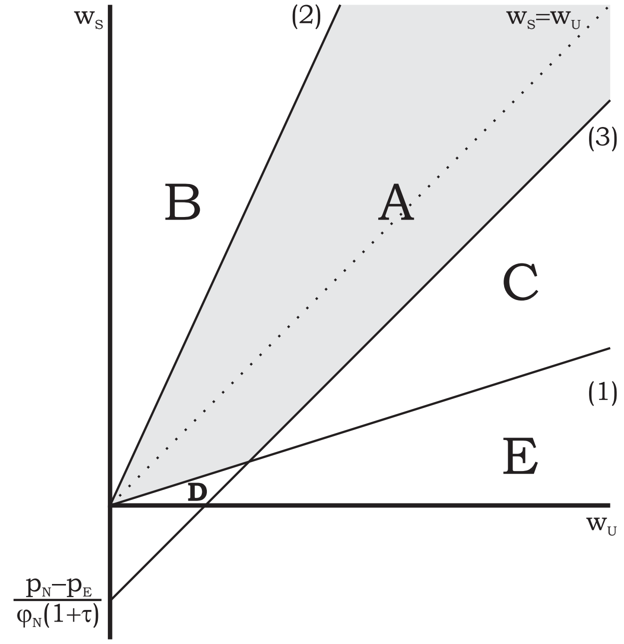
**Proof** We have to show that the set $w_S = w_U$ satisfies every condition. From the previous proof we know $p_E/p_N > 1$ and $(A - 1/\theta_U)/(A-1) \leq 1$, hence a function with slope 1 is in between. Condition (3) has the same slope as $w_S = w_U$ but starts not at zero but at negative ordinate intercept. ∎

Figure 2 illustrates proposition 2.2. All wage–combinations above (1) fulfill condition (1), all combinations below (2) fulfill condition (2) and all relative wages above (3) satisfy condition (3). The following table summarizes in which area which condition holds. A "+" symbolizes that the condition is fulfilled, and a "−" shows, where the condition does not hold.

| **Area** | **Condition (1)** | **Condition (2)** | **Condition (3)** |
|---|---|---|---|
| **A** | + | + | + |
| **B** | + | − | + |
| **C** | + | + | − |
| **D** | − | + | + |
| **E** | − | + | − |

In area A all conditions are satisfied and furthermore it contains the set where both wages are the same, i. e. there is no wage gap—a graphical proof of proposition 2.2. Note that (1) and (2) are only necessary for proposition 2.1 and hence it is valid in areas A, B and C. As was noted earlier, it is possible that $(A - 1/\theta_U)/(A-1)$ turns negative. In this case areas D and E do not exist any more. A higher difference between $p_E$ and $p_N$ increases the slope of (2) and decreases the ordinate intercept of (3) and therefore broadens area A and

Figure 2: Graphical illustration of conditions (1), (2), (3) and proposition 2.2




Source: Own model as described in text.

hence the area where proposition 2.2 is valid. A change in preferences in favor of educated children increases the slope of (1) and hence narrows the area where proposition 2.2 is valid. If time spend on non–educated children increases, (3) shifts up but still remains below the bisecting line and narrowing again the area where proposition 2.2 is valid. This has no effect on the valid areas of proposition 2.1.

Considering this results we are able to give some policy recommendations concerning the qualification structure of its population. However, we made several quite strict assumptions, especially because we consider children as a normal consumption good. Other aspects have to be considered as well, e. g. expectations of income development, children considered as an investment good etc. Our model framework implies that a higher wage gap will lead to more educated children. This request is quite radical, but there are several other mechanisms in our model suited as well (or even) better than this claim.

If politicians want to ensure that high–skilled persons do have more (educated) children, their aim must be to narrow area A and broaden area B and/or C. Both aims can be addressed by lowering the price for education compared to the price for non–educated offspring. Another possibility to broaden area C is to set incentives that people increase the amount of income they spend on children and/or lower the time needed for child–care, e. g. trough an increased supply of kindergartens. Please note that we do not address the demographic problems here, i. e. we do not make a statement if the number of children is enough at all. If the number is to low to ensure population stability or other aims, price decreases as well as the necessary time spent on child–care and –education have to be lowered.

However, we can calculate the number of educated children in the economy. Since we consider condition (1) as quite realistic, we consider only the case when this condition is fulfilled. Define $q$ as the share of $L_S$ who marry a partner of the same skill, and $r$ the share of $L_U$. Further let $s$ be the share of parents with different skill level that decide that the skilled partner should educate their children. Therefore we have $\frac{1-q}{2}L_S$ of $L_SL_S$, $\frac{1-r}{2}L_U$ of $L_UL_U$, $\frac{sq}{2}L_S$ of $L'_SL_U$ and $\frac{(1-s)r}{2}L_U$ of $L_SL'_U$. Of course a necessary condition is $qL_S = rL_U$. From this we can calculate the total numbers of educated, $N_E$, as well as non–educated, $N_N$, children:

$$N_E = \frac{[2-q(2-s)]\,w_S + qsw_U}{2\,[p_E + \varphi_N w_S(1+\tau)]}\tau L_S \tag{13}$$

$$N_N = \frac{r(1-s)w_S + [2-r(1+s)]\,w_U}{2\,([_N + \varphi_N w_S(1+\tau)]}\tau L_U \tag{14}$$

According to specific assumptions the number of educated children is similar to the number of skilled labor in the second period and a share of the non–educated children may also be part of that type of labor (see Kremer and Chen, 2002).

# 3 Optimal population structure

As mentioned in the previous section, wages are determined exogenously. We concluded that relative wages beside the relative price for education are of major importance. It is possible for a region to change relative prices trough different measures. This holds not for wages. For getting an advantage policy–makers therefore have to provide other measures. As for example population policy measures.

In this section we consider the optimal structure of population regarding its skilled/ unskilled share in order to maximize the region's income. We assume a small region that is integrated in the world economy hence wages are determined exogenously by world prices and production technologies. The considered factors $L_S$ and $L_U$ are the only ones available for production. Two goods are produced, $X$ which makes intensive use of skilled labor, and $Y$ which makes intensive use of unskilled labor. Furthermore the standard assumptions of the Heckscher–Ohlin model apply. We assume a Leontieff production technology for both goods,

$$X = \min\{\alpha L_U, (1-\alpha)L_S\}^{\epsilon} \quad \text{and} \quad Y = \min\{\beta L_U, (1-\beta)L_S\}^{\epsilon}, \tag{15}$$

where $\epsilon = 1$ (i. e. constant returns to scale) and $\alpha > \beta$. From this we obtain in the general equilibrium as long as the factor endowment remains in the cone of diversification the following wages:[5]

$$w_S = \frac{(1-\alpha)(1-\beta)(\alpha P_X - \beta P_Y)}{\alpha - \beta} \tag{16}$$

$$w_U = \frac{\alpha\beta\left[(1-\beta)P_Y - (1-\alpha)P_X\right]}{\alpha - \beta} \tag{17}$$

where $P_X$ and $P_Y$ are the integrated world equilibrium prices of $X$ respectively $Y$. However, this production technology does have some nice effects on factor wages outside the

[5] See Appendix.

cone: The abundant factor receives an income of zero (see Leamer, 1998). Therefore, if the region completely specializes in the production of $X$—skilled labor is the abundant factor—, factor wages will be $w_S = 0$ and $w_U = \alpha P_X$ and if the region specializes in the production of $Y$—unskilled labor is the abundant factor, wages will be $w_S = (1-\beta)P_Y$ and $w_U = 0$. The region's GDP with respect to the share of skilled people is

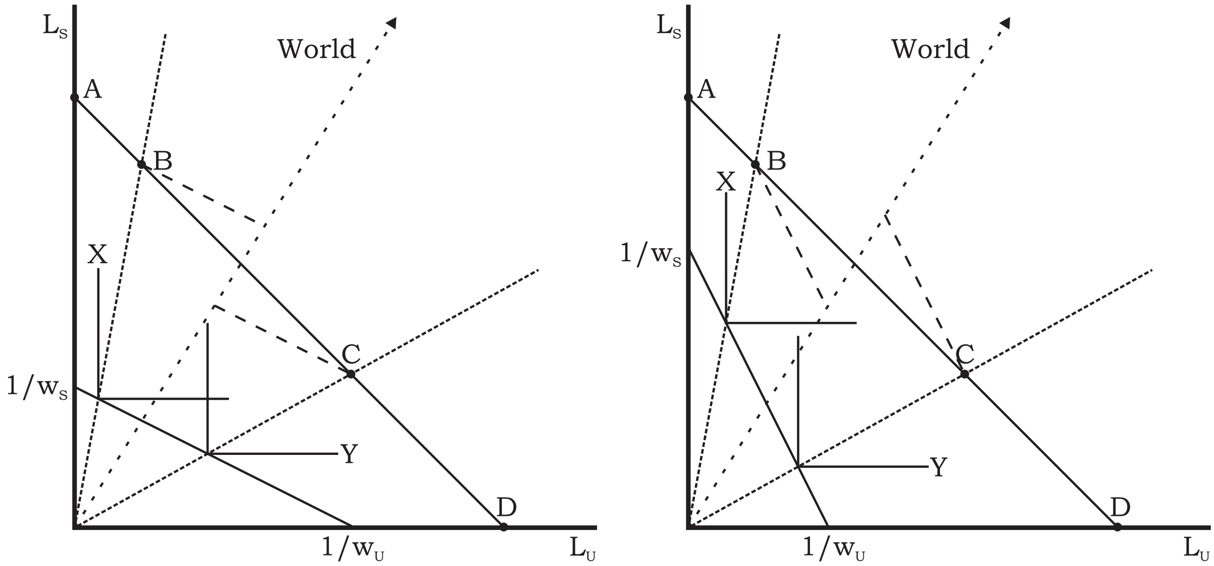
$$GDP = \begin{cases} \varsigma w_S + (1-\varsigma)w_U & \text{no specialization} \\ (1-\varsigma)\alpha P_X & \text{specialization in X} \\ \varsigma\,(1-\beta)\,P_Y & \text{specialization in Y} \end{cases}, \tag{18}$$

where $\varsigma$ is the share of skilled labor of total labor force and

$$\varsigma w_S + (1-\varsigma)w_U = \frac{P_X\alpha(1-\alpha)(\varsigma-\beta) - P_Y\beta(1-\beta)(\varsigma-\alpha)}{\alpha-\beta}. \tag{19}$$

By inspection of (18) we see that maximization will yield a corner solution. Complete specialization in the production of one good, i. e. setting the labor force share $\varsigma$ to zero or one, yields a GDP of zero. The higher is $\varsigma$ the more skilled labor abundant is the region and the higher is the chance that the factor endowment leaves the cone resulting in an income for that factor of zero. The same is true in the other direction. The optimal share must be somewhere in the middle. The best policy hence would try to increase the share of that factor that is worldwide the scarce factor. This information can be gained via the prices (of course (worldwide) preferences do matter as well). E. g. if skilled labor is the worldwide scarce factor, the price of $X$ would be higher than that of $Y$.

Figure 3: Lerner–Diagram




Source: Own model as described in text.

**Proposition 3.1** *Iff $P_X/P_Y > \beta(1-\beta)/(1-\alpha)$ is fulfilled, then the optimal, GDP-maximizing share $\varsigma$ of skilled labor in this region would be $\alpha$ otherwise it would be $\beta$.*

**Proof** Expressions of the output expansion paths are given by

$$L_S = \frac{\alpha}{1-\alpha} L_U \text{ for } X \text{ and } L_S = \frac{\beta}{1-\beta} L_U \text{ for } Y.$$

If we set total population to 1, we get $L_S + L_U = 1$. The two intersection points between this constraint and the output expansion paths are $[1-\alpha, \alpha]$ (denoted $B$ in figure 3) and $[1-\beta, \beta]$ (denoted $C$). Total income in $B$ is $\alpha w_S + (1-\alpha) w_U = \alpha(1-\alpha)P_X$ an in $C$ is $\beta w_S + (1-\beta) w_U = \beta(1-\beta)P_Y$. If total income in $B$ is larger than in $C$ the described condition results. ▮

Please note that the condition stated in proposition 3.1 implies that $w_S/w_U > 1$. You can prove this either by comparing (16) with (17) or by an analysis of figure 3 by comparing the slope of the labor constraint AD and of the isocost line. "World" describes the world endowment vector. Parallels to the isocost line through B and C allow us to interpret the intersection with the world vector as the share of this region on global GDP (see Helpman and Krugman, 1985). We see in the left figure, where $w_S > w_U$ B would yield a higher GDP than C, whereas in the left figure ($w_S < w_U$) C would yield an higher outcome. In both cases A and D would yield an outcome of zero. Only in the case $w_S = w_U$ there are several solutions and the optimal share would be in the interval $[\beta, \alpha]$. If wages are different the region produces only a positive amount of one good and nothing of the other one. But in comparison to the area outside the cone both production amounts are non–negative. In the case of $w_S = w_U$ both amounts are non–negative or positive depending on which solution is chosen.

What do this results imply for our analysis? We saw that if $w_S > w_U$ a higher share of the population $\alpha$ (not the whole!) should be skilled. This case may arise naturally since this wage combination is located in the area B in figure 2. If the share further increases, $w_S$ would drop to zero and eventually E arises lowering the share. If wages for all skill–levels are the same, the share of skilled labor tends to decrease (cf. proposition 2.2). policy–makers have to set prices for education and measurements aiming at decreasing time needed for children so, that the optimal share arises. If the optimal share is reached they must try to change their policies in order to avoid an overshooting. Education policy in a global context becomes a complex and hard to manage task.

# 4 Conclusion

The decision for children depends on a variety of parameters. Important factors are the income of the family as well as the costs of rising them up. In our model these costs change depending on the quality of education of the children, the skill–level of the educating parent and relative wages and hence time–costs. In our model one family member is responsible for child–education. The education–quality outcome hence varies. A skilled parent is more productive in the education of schooled children compared to an unskilled parent. Under (quite) general specifications, a situation arises, where a skilled parent responsible for education has numeral less but more educated children than an unskilled parent. Further of importance is the composition of the family. Whether both partners belong to the same qualification level or to different levels changes results and leads to interesting cases.

If a policy–maker wants to change the education–outcome several measurements are available. Either he changes the direct costs of children (by subsidizing the child–price) or he helps lowering the time–costs of the parent. A change in the composition of the skilled/ unskilled ratio may be advantageous considering an integrated economy. Since wages are determined exogenously, the only measurement available for policy to change GDP and hence welfare, is to change the skilled/ unskilled ratio described previously (under the realistic assumption that (educated) children of skilled parents will be skilled as well). The policy–maker should try to increase the worldwide scarce factor in the region. In the optimum the region is specialized in the production of one good in the sense that the optimal production of the other good is zero, since otherwise the regions income is not maximized.

# A Appendix

We get this wages by using the steps resulting from the analysis of the Lerner-Diagram[6]. We first calculate the unit–value isoquants:

$$P_X X = 1 \Longleftrightarrow \min\{\alpha L_U, (1-\alpha) L_S\} = 1/P_X$$

[6]For a comprehensive analysis of this concept see e. g. Deardorff (2002, 2006)

From this we get the "edge" coordinates and hence the efficient factor input combination to produce a value of one:

$$L_U = 1/\left(\alpha P_X\right) \quad \text{and} \quad L_S = 1/\left((1-\alpha)P_X\right)$$

From the resulting coordinates $[1/\left(\alpha P_X\right), 1/\left((1-\alpha)P_X\right)]$ and $[1/\left(\beta P_Y\right), 1/\left((1-\beta)P_Y\right)]$ we are able to calculate the unit isocost line. We can calculate the slope

$$\Delta = -\frac{\alpha\beta\left((1-\beta)P_Y - (1-\alpha)P_X\right)}{(1-\alpha)(1-\beta)\left(\alpha P_X - \beta P_Y\right)}$$

We know the general formulation of the unit isocost line, $L_S = 1/w_S - w_U/w_S L_U$ and so we are able to calculate the wages:

$$w_S = \frac{(1-\alpha)(1-\beta)(\alpha P_X - \beta P_Y)}{\alpha - \beta}$$

$$w_U = \frac{\alpha\beta((1-\beta)P_Y - (1-\alpha)P_X)}{\alpha - \beta} \qquad \blacksquare$$

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