The Demand for Money by Private Firms in a Regulated Economy

Theoretical Underpinnings and Empirical Evidence for Germany 1960 – 1998

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Content

Abstract

Based on a cash-in-advance approach, this paper investigates theoretically the determinants of money holdings of firms under the conditions of a highly regulated labor market and analyses empirically the demand for money of German businesses during the period 1960–1998. As a result of our theoretical analysis the demand for cash balances by firms for shadow market activities depends among other things positively on the expected wage wedge. The empirical results show that the coefficient of the wage wegde has a positive sign in the long-run cointegrating relationship and is statistically significant positive in the short-run dynamics of the error correction model.

Key words: Money Demand by Firms, Wage Wedge, Cash-in-Advance Model, Cointegration, Error-Correction

Auf der Grundlage eines Cash-in-advance-Ansatzes untersucht der vorliegende Beitrag die Bestimmungsgründe der Geldnachfrage von deutschen Unternehmen (1960–1998) – vor dem Hintergrund eines hoch regulierten Arbeitsmarktes. Das theoretische Modell ergibt, daß Unternehmen Kasse für Aktivitäten auf dem Markt für Schwarzarbeit unterhalten und zwar um so mehr, je größer die Kluft zwischen den Bruttoarbeitskosten und den Nettolöhnen ("wage wedge") ist. Der Koeffizient der "wage wedge" weist ein positives Vorzeichen in der Kointegrationsbeziehung auf und ist statistisch signifikant positiv in der kurzfristigen Dynamik des Fehler-Korrektur-Modells.

Schlagworte: Geldnachfrage von Unternehmen, Cash-in-advance-Modell, Kointegration, Fehler-Korrektur-Modell, Lohnzusatzkosten

JEL-Klassifikation: E41, C22, J30

1 INTRODUCTION[1]

While empirical investigations into the properties of money demand have tended to throw their efforts on the aggregate level, the famous "finance motive" of money holdings put forward by Keynes (1936) and the early theoretical investigations by Baumol (1956) and Miller and Orr (1966) using an inventory-theoretic framework seem to point more at the business sector than at the aggregate level. More recently, there have been only a few remarkable studies on the money demand of firms (Fase and Winder 1990; Barr and Cuthbertson 1992; Mizen 1996; Viren 1996; Mulligan 1997). As it appears to us, none of the authors mentioned, however, has attempted to link money demand of the business sector to the demand for labor on informal, likewise grey labor markets in the shadow economy.

It is a matter of daily observation in highly industrialized economies that firms are inclined to hire work from informal markets whenever the burden of labor costs – in addition to the wage rate before taxes – makes this alternative profitable vis-à-vis the formal labor market. The sector of housing and construction is a typical and perhaps the most prominent example where companies draw heavily on labor input from informal sources and do hold cash exactly for this purpose. We are not so much interested – as some of the mentioned papers are – in the existence of economies of scale in the demand for cash by firms. Moreover, our main focus is on the determinants of money holdings by firms under the conditions of a highly regulated economy, especially in the labor market.

In glancing through the literature, it turned out that for the attempt to model firms' money demand in such an environment, much can be learned from a body of literature interested in – at first sight – quite a different issue, namely the demand for money in an economy with various constraints (Lane 1990a, 1990b, 1992). In these papers, Lane models households as choosing optimal money holdings subject to constraints in formal and informal goods as well as foreign exchange markets during the socialist experiment. Also, Lane incorporates the precautionary motive of money holdings into his analysis. In the firms' view of our own work, this can be translated into a precautionary finance motive on the background of informal labor supply. The cash-in-advance approach chosen by Lane will be used, here, as well. Recently, Bohl and Sell (1998) have done a similar methodological exercise for the demand for currency in an open economy.

The paper is organized as follows. In the following chapter 2, our own model will be presented. In section 3, the derived money demand function is subject to an empirical test with data from Ger-

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many applying modern econometric techniques. Thereafter, we will give a brief review on the empirical results achieved by other studies on the money demand of firms in section 4. Finally, in section 5, we give a summary of our results and some hints as to future research necessities.

2 THE MODEL

The following analysis is not only a full application, but also an extension of Lane's (1992) cash-in-advance model for the household money demand in Poland to firms' money demand in a market economy with a significant shadow economy. A representative firm maximizes expected output, which is a function of labor input, over an infinite time horizon. The firm holds two cash balances, both in domestic money. Purchases of labor input must be backed with money. Apparently, this assumption only applies to a minority of sectors in a modern economy and, in the past, it could be most likely found in the services and/or in the construction sector. For simplicity, it is assumed that labor is the only factor of production. The price for labor is lower in the shadow economy, but the quantity available there is limited and uncertain, whereas in the official market, the firm may hire labor as much as desired, but at a higher price. Therefore, domestic currency is held in preparation for hiring labor at a favorable price on the parallel labor market.[2]

The timing of transactions in the model is as follows. In the morning, the firm hires workers in the two markets, constrained by money held over from the previous day. In the afternoon, the firm's production is sold. There are no other financial assets. The price and the available quantity of labor in the shadow economy, as well as the price of labor in the official market, are treated as random variables, whose realization is not known until the beginning of the day.

The firm maximizes expected output y, which is a function of factor input, i_t, over an infinite horizon:

$$(1) \qquad \max \; EO = E_0 \sum_{t=0}^{\infty} b^t y(i_t) \rightarrow Max!$$

where E_t denotes the expectations operator conditional on information available at time t. b is a discount factor and EO stands for expected output. For the sake of simplicity, factors of production only consist of labor, which is hired either on official or on parallel markets:

$$(2) \qquad i_t = f_t^o + f_t^s$$

where f_t^o and f_t^s is the amount of labor hired in the official market and the shadow economy, respectively. Maximizing output, as stated in equation (1), is, according to duality theory (Nicholson

1992), equivalent to the minimization of costs for a given (but not necessarily constant over time) total cost of inputs ($TC_t = f_t^o l_t^o + f_t^s l_t^s$). Insofar, our approach is capable to explore the optimal production plan of the firm. In addition, we have a view on the optimal financial plan of the firm: the firm's financial budget constraint for each period requires that expenses for labor during the period plus end-of-period money holdings are financed by money carried over from the previous period and by sold output:[3]

(3) $$f_t^o l_t^o + f_t^s l_t^s + m_t^{d_o} + m_t^{d_s} = m_{t-1}^{d_o} + m_{t-1}^{d_s} + y_t$$

where l_t^o denotes the price of labor on the official market, l_t^s the price of labor in the shadow economy, $m_t^{d_o}$ the money holdings for the official market and $m_t^{d_s}$ the money holdings for the shadow market.

From first glance, the distinction between two different cash holdings – one legal and one illicit – of the same currency by a representative firm may seem to be artificial. But one should be aware of the fact that legal cash belongs to the official assets of the firm and, hence, enters the firm's books while illicit cash never does. Moreover, in making the distinction between these two components of currency demand, we follow the procedure of Bhattacharyya (1990). As opposed to Bhattacharyya, however, here both components and its economic determinants will be derived from optimality conditions in a constrained maximisation problem. Expenses for hiring labor in the shadow economy are limited by holdings of illicit cash balances carried over from the previous period:

(4) $$f_t^s l_t^s \leq m_{t-1}^{d_s}$$

Previous money holdings for the shadow labor market can hence either be interpreted as an exhaustible resource which diminishes over time or as a variable which is exogenous to the model![4] Total expenses for hiring labor on the two markets are constrained by the two types of cash balances:

(5) $$f_t^o l_t^o + f_t^s l_t^s \leq m_{t-1}^{d_o} + m_{t-1}^{d_s}$$

The amount of labor that the firm can hire in the shadow economy is taken to be limited by the availability of supply. The risk for that labor in the parallel market is to be detected, rises with the

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2 See Sell (1997) for an analysis of the income velocity of money in the informal sector.

3 For a constant price level of one, production equals sold output: $p_t y_t = Y_t$ for $p_t = 1$.

4 There are a number of possibilities to endogenize the process of replenishing illicit cash balances. For instance, we could argue that output (see equation three above) can be sold on official as well as on parallel markets, the latter

unofficial wage rate, because authorities will amplify controls when observing the latter to increase. Hence, the wage rate is lower than the one which would equal demand and supply on the parallel labor market and, hence, labor demand is rationed on the parallel market:

(6) $$f_t^s \leq \bar{f}_t^A$$

where the quantity of labor available is $\bar{f}_t^A$. There are also nonnegativity conditions on the holdings of the two cash balances and on the hiring of labor in the two markets:[5]

(7) $$m_t^{d_o}, m_t^{d_s} \geq 0$$

(8) $$f_t^o, f_t^s \geq 0$$

The firm, therefore, maximizes the expected discounted value of production (equation (1)) with respect to the quantities of hired labor (in each market) and to the holdings of legal and illicit cash in each period, subject to the constraints (3) through (8). Defining a value function, $J(m_t^{d_o}, m_t^{d_s})$, as the expected maximized present value of production from period *t + 1* onward as a function of money carried over from period t, the Lagrangean for each period t can be written as follows:

(9)
$$\begin{aligned} L = {} & y\left(f_t^o + f_t^s\right) + bJ\left(m_t^{d_o}, m_t^{d_s}\right) + l_{0,t}\left(TC - f_t^0 l_t^0 - f_t^s l_t^s\right) \\ & - l_{1,t}\left(f_t^o l_t^o + f_t^s l_t^s + m_t^{d_o} + m_t^{d_s} - m_{t-1}^{d_o} - m_{t-1}^{d_s} - y_t\right) \\ & - l_{2,t}\left(f_t^s l_t^s - m_{t-1}^{d_s}\right) - l_{3,t}\left(f_t^o l_t^o + f_t^s l_t^s - m_{t-1}^{d_o} - m_{t-1}^{d_s}\right) \\ & - l_{4,t}\left(f_t^s - \bar{f}_t^A\right) - l_{5,t} m_t^{d_o} - l_{6,t} m_t^{d_s} - l_{7,t} f_t^o - l_{8,t} f_t^s \end{aligned}$$

Now, we define:

(10) $$q_t = \frac{l_t^o - l_t^s}{l_t^o} > 0$$

where q_t is the excess price for labor to be payed by entrepreneurs in the official market relative to the market in the shadow economy. We shall call it the "wage wedge" hereafter. By this we mean (as a percentage rate) the wedge between labor costs and net income of employees (take home pay). The price which accrues from hiring labor in the official market can be calculated according to the compensation of employees concept (Hinze 2000). In this concept it is assumed that firms have to

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implying a good's price free of taxes. The revenue on the parallel goods market, then would serve to replenish illicit cash in order to hire labor on the shadow labor market in the next period.

5 There are three sources for domestic demand for cash balances: legal and illicit cash of firms, cash balances of private households, and, finally, demand for domestic cash from abroad: $m_t^d = m_t^{d_o} + m_t^{d_s} + m_t^{d_{hh}} + m_t^{d_f}$.

afford – in excess of gross wages and salaries – the respective social security payroll tax. Opposed to this, the employees' utility reservation wage level corresponds to the net income of employees (wages/salaries) after deduction for personal income tax and social security contributions. The price for hiring labor in the shadow economy equals in equilibrium by and large the utility reservation wage level from the official economy.

Using this device, one obtains the following first order conditions for hiring labor in the two markets:

(11) $$\frac{\partial y}{\partial f_t^{o}} = l_t^{o}\left(\lambda_{0,t} + \lambda_{1,t} + \lambda_{3,t}\right) + \lambda_{7,t} = \frac{\partial y}{\partial i_t} = y_t^{'}$$ because of (2)

(12) $$\frac{\partial y}{\partial f_t^{s}} = l_t^{s}\left(\lambda_{0,t} + \lambda_{1,t} + \lambda_{2,t} + \lambda_{3,t}\right) + \lambda_{4,t} + \lambda_{8,t} = \frac{\partial y}{\partial i_t} = y_t^{'}$$ because of (2)

where the $\lambda_{i,t}$ $(i = 1,...,8)$ are static Lagrangean multipliers. Equations (11) and (12) are the first order conditions for hiring labor in the two markets. As in Lane (1992) we assume, for simplicity, that some labor is hired in each of the two labor markets in each period so that $\lambda_{7,t} = \lambda_{8,t} = 0$. Otherwise, the tax authorities could anticipate a production in the shadow economy. Hence, conditions (11) and (12) can shortened to become:

(13) $$\frac{\partial y}{\partial i_t} = y'_t = l_t^{o}\left(\lambda_{0,t} + \lambda_{1,t} + \lambda_{3,t}\right)$$

(14) $$\frac{\partial y}{\partial i_t} = y'_t = l_t^{s}\left(\lambda_{0,t} + \lambda_{1,t} + \lambda_{2,t} + \lambda_{3,t}\right) + \lambda_{4,t}$$

Conditions (13) and (14) can be combined to yield:

(15) $$y'_t = l_t^{s}\left(\lambda_{0,t} + \lambda_{1,t} + \lambda_{2,t} + \lambda_{3,t}\right) + \lambda_{4,t} = l_t^{o}\left(\lambda_{0,t} + \lambda_{1,t} + \lambda_{3,t}\right)$$

which, solved for $\lambda_{2,t}$, gives:

(16) $$\lambda_{2,t} = \frac{l_t^{o} - l_t^{s}}{l_t^{s}}\left(\lambda_{0,t} + \lambda_{1,t} + \lambda_{3,t}\right) - \frac{\lambda_{4,t}}{l_t^{s}}$$

Introducing (16) into (14), we get:

(17) $$y'_t = l_t^{s}\left(\lambda_{0,t} + \lambda_{1,t} + \frac{l_t^{o} - l_t^{s}}{l_t^{s}}\left(\lambda_{0,t} + \lambda_{1,t} + \lambda_{3,t}\right) + \frac{\lambda_{4,t}}{l_t^{s}} + \lambda_{3,t}\right)$$

Rearranging (16) gives (18):

(18) $$\left(l_{2,t} + \frac{l_{4,t}}{l_t^s}\right)\left(\frac{l_t^s}{l_t^o - l_t^s}\right) = \left(l_{0,t} + l_{1,t} + l_{3,t}\right)$$

Then, (18) introduced into (13) gives:

(19) $$y'_t = \left(l_{2,t} + \frac{l_{4,t}}{l_t^s}\right)\left(\frac{l_t^s}{l_t^o - l_t^s}\right) l_t^o$$ or

(20) $$y'_t = \left(l_{2,t} l_t^o + l_{4,t}\frac{l_t^o}{l_t^s}\right)\left(\frac{l_t^s}{l_t^o - l_t^s}\right)$$ or, finally

(21) $$y'_t = \left[l_{2,t} l_t^o + l_{4,t}\left(1 + \frac{l_t^o - l_t^s}{l_t^s}\right)\right]\left(\frac{l_t^s}{l_t^o - l_t^s}\right)$$

Because of:

(22) $$\frac{q_t}{(1 - q_t)} = \frac{l_t^o - l_t^s}{l_t^s}$$

we can rewrite (21) to become:

(23) $$y'_t = \left[l_{2,t} l_t^o + l_{4,t}\left(1 + \frac{q_t}{1 - q_t}\right)\right]\left(\frac{1 - q_t}{q_t}\right)$$

Rearranging (23) leads to (24):

(24) $$\frac{q_t}{1 - q_t} = \frac{l_{2,t} l_t^o + l_{4,t}}{y'_t - l_{4,t}}$$

The wedge between the prices of labor in the official and in the parallel market is either associated with shortages in the parallel labor market ($l_{4,t}$) or with a lack of money to pay labor there ($l_{2,t}$). In other words, given the low prices but limited quantities in the parallel labor market, firms either hire everyone who is available at the low wage costs (still taking into account the non-negativity condition (8)), or run out of illicit cash in the attempt.

Next, consider the conditions for optimal holdings of legal and illicit cash:

(25) $$\frac{\P L}{\P m_t^{d_o}} = b\,\frac{\P J}{\P m_t^{d_o}} - l_{1,t} - l_{5,t} = 0\,;$$

(26) $$bJ_{d_o,t} = l_{1,t} + l_{5,t}$$

(27) $$\frac{\P L}{\P m_t^{d_s}} = b\,\frac{\P J}{\P m_t^{d_s}} - l_{1,t} - l_{6,t} = 0\,;$$

(28) $$bJ_{d_s,t} = l_{1,t} + l_{6,t} \quad \text{where}$$

(29) $$J_{i\,,t} = \frac{\P J\,(\,m_t^{d\,o}\;,m_t^{d\,s}\;)}{\P m^{i\,t}}\,; \qquad i = d_0\;,d_s$$

Using the definition of the value function:

(30) $$J_{d_o,t} = \frac{\P J}{\P m_t^{d_o}} = E_t\left(l_{1,t+1} + l_{3,t+1}\right)$$

(31) $$J_{d_s,t} = \frac{\P J}{\P m_t^{d_s}} = E_t\left(l_{1,t+1} + l_{2,t+1} + l_{3,t+1}\right)$$

That is, legal cash is expected to contribute to future production by easing the budget constraint (reflected in $l_{1,t}$) as well as to the extent that it may ease a cash-in-advance constraint ($l_{3,t}$). Holding illicit cash is expected to ease the budget constraint ($l_{1,t}$) and two cash-in-advance constraints ($l_{2,t}$, $l_{3,t}$). In the mathematical annex, we show how the four mentioned first order conditions can be combined to yield a proxy for the determinants of illegal and legal cash. Namely, in equation (57) – see annex – we find a formulation for the relative attractiveness to hold illicit rather than legal cash.

Equation (59),[6] which is a second-order Taylor approximation of equation (57), can then be used to provide a portfolio-theoretic rationale for the (il)legal demand for domestic currency. First, the demand for (il)legal cash depends (positively) negatively on the expected excess price of labor in the official market over the price of labor in the parallel market. This labor price premium is divided by the expected parallel market wage increase factor, which deflates the expected labor price premium. Second, the higher the parallel market wage increase factor, the (lower) higher is demand for (il)legal cash, ceteris paribus. Demand for (il)legal cash depends on the variances and covariances of labor input and wage inflation on the parallel market. The implication of the variance of labor input is ambiguous and depends on the third derivative of the production function y’’’. Third, a higher variance of wage inflation on the parallel market is associated with a (higher) lower demand for (il)legal cash. Also, the demand for (il)legal cash is (negatively) positively related to the covariance between expected labor input and the expected wage premium of the official over the parallel market. In the same vein, the demand for (il)legal cash is (positively) negatively associated with the

6 See the mathematical annex for a full and step by step derivation of equation (59).

covariance between expected labor input and wage inflation on the parallel market. Fourth, the demand for (il)legal cash is (negatively) positively related to the covariance between wage inflation on the parallel market and the wage premium of the official over the parallel market. Taking into consideration the role of output in equation (3) and the fact that labor input is not an observable variable, the demand for (il)legal cash can be expressed as a function of the following five determinants:

(32) $$m_t^{d_o} = m\left(\overset{+}{y}_t, E_t\overset{-}{q}_{t+1}, E_t\overset{+}{p}_t^{\,s}, \operatorname{var}\overset{-}{p}_t^{\,s}, \overset{+}{\operatorname{cov}(q_{t+1}, p_t^{\,s})} \right)$$

Hence, mutatis mutandis demand for illicit cash reads:

(33) $$m_t^{d_s} = m\left(\overset{+}{y}_t, E_t\overset{+}{q}_{t+1}, E_t\overset{-}{p}_t^{\,s}, \operatorname{var}\overset{+}{p}_t^{\,s}, \overset{-}{\operatorname{cov}(q_{t+1}, p_t^{\,s})} \right)$$

It comes as no surprise that neither legal nor illicit cash depend on the interest rate or on any other opportunity cost variable – given that there are no other financial assets than money (see above) in the model.[7] As both components of currency demand are not observable individually, there seems to be a problem to estimate a currency demand of firms equation. As only production enters into both components/demand functions with the same sign and both demand functions are symmetric in all the other variablers with regard to the sign, one way out would be to estimate a cash demand by firms function, where only production enters as a determinant. The disadvantage of doing so, however, would consist in losing all the additional information embedded in the components (32), (33). Hence, a second way out consists in building the aggregate as in Bhattacharyya (1990):

(34) $$\begin{aligned} m_t = m_t^{d_s} + m_t^{d_o} = \left(a_{d_s} + a_{d_o}\right) y_t + \left(b_{d_s} - b_{d_0}\right) E_t q_{t+1} + \left(g_{d_o} - g_{d_s}\right) E_t p_t^{\,s} \\ + \left(d_{d_s} - d_{d_o}\right) var p_t^{\,s} + \left(e_{d_o} - e_{d_s}\right) cov\left(q_{t+1}, p_t^{\,s}\right) \end{aligned}$$

If the illicit (official) component dominates, then the estimated five compound coefficients in (34) should have the following signs: $a, b, d > 0$ (<0), but $g, e < 0$ (>0).

3 DATA AND EMPIRICAL RESULTS

The time series used in our study are annual observations covering the period from 1960 to 1998. The monetary aggregate for the business sector consists of cash balances and sight deposits and is from the financial accounts for Germany published by the Deutsche Bundesbank (Deutsche Bundesbank 1994, 1999). The statistics of the Deutsche Bundesbank (Finanzierungsrechnung) do not

7 See for an analogous result Seitz (1995).

allow for a breakdown of money holdings into cash and sight deposits (Brümmerhoff 1985). Therefore, when estimating money demand functions of firms instead of demand for cash balances functions (Bohl and Sell 1998) of these firms, additional motives of holding sight deposits which go beyond the scope of this paper come into play. The other problem – that a breakdown of the sectors', and hence also firms' cash holdings into their (its) different components is usually unavailable – is not a hindering factor in principle for our empirical analysis, as we have shown in the last section. Financial assets and liabilities of firms are derived from the statistics of financial institutions and not from the balance sheets of the firms (Bundesbank 1999). This makes sure that we deal with net financial assets which are not those observed by the fiscal authorities and can be subject of payments to the shadow economy. Information on cash holdings of firms result from consistency rules applied to all gross and net balances for the non-financial sectors and should, hence, cover all (legal and illicit) cash transactions of this sector. We deflate this time series by the GDP deflator to construct real money balances m_t. The GDP deflator is taken from the annual report of the German Council of Economic Experts (Sachverständigenrat 2001). The gross value added of the business sector in constant prices serves as the scale variable y_t.

What about the size of the shadow economy in Germany? Recent calculations by Schneider (1999) say that the income generated in the parallel economy at the beginning of the new century corresponds to approximately 15 % of GDP. Empirical estimates with econometric techniques (Karmann 1986, 1990) show that the share of the shadow economy was almost zero in Germany in 1970. Since then, it has been rising and reached a level of 10 % of GNP already in 1990. If Schneider's recent calculations are correct, the speed of growth of the hidden economy has accelerated during the 1990s.

To construct the wage wedge w_t we divide the total cost per employee (according to the compensation of employee concept) by the net wage/salary per employee. This ratio was 1.376 at in 1960 and has increased meanwhile (1999) to a level of approximately 1.934. Between 1960 and 1970 the ratio remained rather stable. Since 1970, however, the ratio has been ascending considerably. This finding matches nicely the behavior of the shadow economy (see above). Only since the end of the 1990s we do experience a relaxation in the speed of increase. The time series on which we build our empirical analysis are from the Hamburg Institute of International Economics (Hinze 2000).

The variables m_t, y_t and w_t are in logarithms and refer from 1990 onwards to West and East Germany. To use actual rather than expected values of the wage wedge, we rely on the argument put forward by Taylor (1991), according to which the application of cointegration techniques allows to test our money demand function subject only to the very weak assumption that forecasting errors are stationary.

To investigate the degree of integration of the times series we employ the test suggested by Kwiatkowski et al. (1992), hereafter KPSS test. In the KPSS test the null hypothesis of the stationarity of a variable is tested against the alternative of a unit root. The results of these tests for the null hypothesis of level stationarity and trend stationarity employing the truncation lags from 0 to 3 are reported in Table 1 where the maximum lag length is chosen due to the suggestion in Schwert (1987) l = int $\{4(T/100)^{1/4}\}$. The KPSS tests are performed for the variables m_t, y_t and w_t assuming that the other variables in our money demand equation are stationary time series. As can be seen in Table 1, the null hypotheses of level and trend stationarity are rejected for all three time series because the test statistics are significant at the five per cent level. Hence, we assume that m_t, y_t and w_t are integrated of order one.

Next, we estimate and test for cointegration relying on the Johansen procedure (Johansen 1988, 1991) and the Engle-Granger approach (Engle and Granger 1987) between the instationary time series. Panel A in Table 2 presents the tests of the null hypothesis of r cointegrating vectors using the trace test. The test statistics show the existence of a single cointegrating relationship because the null hypothesis of zero cointegrating vectors is rejected at the five per cent level while the hypothesis that the number of cointegrating vectors is less than or equal to one cannot be rejected. The underlying VAR model has the lag length one and the LM-type tests (LM1, LM4) show the absence of first and fourth order autocorrelation. When looking at the estimated cointegrating vector (after normalizing on m_t) the long-run income elasticity is close to one and the coefficient of the wage wedge has a positive sign. Additional insights into the cointegrating relationship between the three variables can be obtained by examining the results of the Engle-Granger approach in Panel B. Cointegrating Durbin-Watson (CRDW) as well as cointegrating Dickey-Fuller (CRDF) test statistics are significant and reject the null hypothesis of no cointegration. Compared with the findings of the Johansen procedure in Panel A the estimated coefficient of the scale variable is again close to one and the estimated parameter of the wage wedge has a positive sign.

Having analysed the stochastic properties of the individual time series and the long-run relationship we have estimated an error correction model which includes all explanatory variables of our money demand function. Starting with this general specification we have excluded step by step the variables with statistically insignificant coefficients at the five per cent level. The result of this general-to-specific testing strategy is the following error correction model:

$$Dm_t = \underset{(2.56)^*}{0.06} + \underset{(2.66)^*}{1.47\Delta w_{t-1}} - \underset{(2.09)^*}{0.31\hat{u}_{t-1}} + \hat{e}_t$$

$DW = 2.03$ $Q_{(7)} = 4.99$ $Q_{(3)} = 1.85$ $\overline{R}^2 = 0.45$ $RESET = 0.77$

Period: 1962 – 1998 $u_t = m_t + 3.10 - 1.01 y_t - 1.97 w_t$

The model contains as explanatory arguments the stationary long-run relationship between real money, the scale variable and the wage wedge and in addition only the lagged wage wedge as an explanatory variable of the short-run dynamics. The parameter on the wage wedge has a positive sign. All other arguments of our money demand model were statistically insignificant. This result is not contradictory to the implications of the model (see equations (32) and (33)). As the statistics of business cash balances and sight deposits are not calculated from the firms‘ books, but from sources of the consolidated monetary sector, they represent a real "mix" of legal and illicit cash motives. A positive sign of the wage wedge in the above estimation equation, hence, is a signal for a significant (and dominant) influence of the parallel labor market on the money holdings of the firms. The coefficient of the error correction term is statistically significant different form zero which confirms the findings of the cointegration analyses. Furthermore, the residuals of the error correction model exhibit no autocorrelation as can be seen by the Durbin-Watson (DW) and Ljung-Box ($Q_{(df)}$) tests. In addition, the RESET test shows that there is no functional misspecification. Measured by the adjusted coefficient of determination ($\overline{R}^2$) the explanatory power of the model is acceptable. In summary, the error correction model has statistically and economically sensible characteristics and describes the behavior of the demand for money quite well.

4 FINDINGS OF RELATED STUDIES AND COMPARISON WITH OUR RESULTS

As mentioned in the introduction, our study is part of the small literature on the money demand of firms so that it is sensible to review this branch of studies and compare their findings with our own results. Meltzer (1963), Whalen (1965) and Vogel and Maddala (1967) wrote the first articles investigating the behaviour of demand for money of companies and stimulated subsequent research activities. More recently, Fase and Winder (1990) specify error correction models to study aggregate and sectorally disaggregated money demand functions for M1 and M2 in the Netherlands over the period from 1970 to 1988. An income elasticity of unity was found for the business sector, the household and the aggregate money demand functions irrespective of the money definition used. The interest and inflation elasticities for the business sector are systematically higher in absolute terms than the values of the household sector money demand functions and the aggregate demand for money functions. Again, these findings are insensitive to the usage of different money definitions and could be a result of the greater scope which the business sector has for conducting active and systematic cash management compared with the household sector. In addition, Fase and Winder experiment with a cyclical indicator as part of the short-term dynamics and as a proxy for the costs of real assets for which the liquid financial assets are a substitute. The coefficients of this variable in all money demand functions for the business sector are statistically insignificant different from zero.

Barr and Cuthbertson (1992) analyse the company sector liquid asset holdings in the United Kingdom within a system framework for the period from 1976 to 1986. They implement cointegration techniques and obtain asset demand functions which satisfy the theoretical restrictions. The estimated demand functions are intuitively plausible and exhibit parameter stability. The demand of the company sector for liquid assets depends on various rates of return and on wealth. In Mizen (1996), a forward-looking buffer stock model for the company sector in the United Kingdom over the period from 1970 to 1988 is specified. The results suggest that the buffer stock models for the money demand behaviour of the company sector are validated and that businesses hold money for the insulation against unanticipated shocks. Compared with the findings from studies on aggregate money demand functions the long-run cointegrating relationships reflect a different behaviour of the company sector, their money holdings are far more volatile and may well be interrelated with bank lending and net trade credit arrangements. These results are not surprising because the company sector actively monitors and adjusts its balances far more often than the personal sector.

Among the recent investigations on the demand for money of companies there are two cross-sectional studies which should be mentioned. Mulligan (1997) provides a cross-sectional study of the demand for money relying on 12,000 firms in the United States for the period from 1961 to 1992. The money demand function considered relates money holdings to the volume of sales, the opportunity cost of holding money and the value of the cash manager's time. His findings indicate that there are economies of scale and that companies headquartered in countries with high wages hold more money for a given level of sales. The estimated scale as well as wage elasticities are by and large of equal amount and lie around 0.8 and the interest elasticity is statistically significant negative. Viren (1997) analyses the relationship between the demand for cash and the transactions volume by 2,700 Finnish firms. Relying on this cross-section of business firms the findings show economies of scale and that the relationship of demand for cash and the transaction volume differs across the branches of the economy. Cash is not predominantely used in the shadow economy and is still a competitive means of payment in the service sector of the Finnish economy.

The review of the existing literature on the money demand of firms demonstrates that none of the studies has investigated theoretically nor empirically the importance of the demand for labor in the shadow market for the firms' money demand. The investigations rely mostly on the inventory theoretical model put forward by Baumol (1956) and Tobin (1956), while our own estimated money demand function is based on Lane's (1992) cash-in-advance model. The findings of the studies mentioned above can be reconciled with our own results concerning long-run elasticity of the scale variable because the estimated values are comparable. To our knowledge there is no investigation on the demand for money of firms for Germany. Virtually all studies analyse the aggregate money demand for M1 and M3 (see, for example, Falk and Funke 1995, Hansen and Kim 1995 and Wolters et al. 1998). Comparing their findings with our results, it is interesting to note that the long-run elasticity of the scale variable is generally well above one, while the estimated long-run elasticity of the scale variable in the firms' money demand is near one. When looking at the error correction model the estimated parameter of the error correction term in our model is higher in absolute terms. This indicates faster adjustment towards equilibrium on the firms' level than on the aggregate level.

5 SUMMARY

Based on Lane's (1992) cash-in-advance approach, the paper investigates theoretically the determinants of money holdings of firms under the conditions of a highly regulated labor market and analyses empirically the demand for money of German businesses during the period from 1960 to 1998. The paper differs from previous studies because none of these has attempted to link money demand of firms to the demand for labor in the shadow economy. Moreover, there are no empirical investigations concerning the demand for money of German firms.

As a result of our theoretical analysis the demand for cash balances by firms for shadow market activities depends among other things positively on the expected wage wedge, which is the excess price for labor to be paid by firms in the official market relative to the price to be paid in the shadow economy. Relying on this theoretical finding, we concentrate our interest on the importance of the wage wedge as an explanatory argument of a money demand function for German firms. The empirical results support the theoretical findings in the sense that the coefficient of the wage wegde has a positive sign in the long-run cointegrating relationship. Furthermore, the wage wedge appears with a statistically significant positive parameter in the short-run relationship of the error correction model, which captures the money demand behavior of German firms quite well. Hence, the results point at the relevance of illicit cash as a part of firms' money demand during the period under analysis.

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MATHEMATICAL ANNEX

Equation (30) inserted into (26) and (31) inserted into (28) gives:

(35) $$bE_t\left(l_{1,t+1} + l_{3,t+1}\right) = l_{1,t} + l_{5,t}$$

(36) $$bE_t\left(l_{1,t+1} + l_{2,t+1} + l_{3,t+1}\right) = l_{1,t} + l_{6,t}$$

Equations (35) and (36) can be simplified by assuming that $l_{3,t} = l_{6,t} = 0$. In this case, illicit cash is also used as a long term store of value and the firm thus does not exhaust its illicit cash this period (later, we will proceed in the same way with regard to legal cash). Hence, neither the overall cash-in-advance constraint (equation (5)) nor the nonnegativity constraint on illicit cash balances (equation (7)) is binding in period *t*. Solving (13) and (14) for $l_{1,t}$ (and for $\lambda_{1,t} + \lambda_{2,t}$, respectively) leads to (38) and (40) respectively:

(37) $$y'_t = l_t^o(l_{0,t} + l_{1,t})$$

(38) $$\frac{y'_t}{l_t^o} = l_{0,t} + l_{1,t};\ l_{1t} = \frac{y'_t}{l_t^0} - l_{0,t}$$

(39) $$y'_t = l_t^s\left(l_{0,t} + l_{1,t} + l_{2,t}\right) + l_{4,t}$$

(40) $$l_{0,t} + l_{1,t} + l_{2,t} = \frac{y'_t - l_{4,t}}{l_t^s};\ l_{1,t} + l_{2,t} = \frac{y'_t - l_{4,t}}{l_t^s} - l_{0,t}$$

Applying (40) to the next period and inserting into (36) gives:

(41) $$E_t\left(l_{1,t+1} + l_{2,t+1}\right) = E_t\left(\frac{y'_{t+1} - l_{4,t+1}}{l_{t+1}^s}\right) - E_t\left(l_{0,t+1}\right)$$

and, extending to:

(42) $$bE_t\left(l_{1,t+1} + l_{2,t+1}\right) = bE_t\left(\frac{y'_{t+1} - l_{4,t+1}}{l_{t+1}^s}\right) - bE_t\left(l_{0,t+1}\right)$$

Hence,

(43) $$bE_t\left(\frac{y'_{t+1} - l_{4,t+1}}{l^s_{t+1}}\right) - bE_t\left(l_{0,t+1}\right) = l_{1,t}$$

Introducing (38) into (43) yields:

(44) $$bE_t\left(\frac{y'_{t+1} - l_{4,t+1}}{l^s_{t+1}}\right) - bE_t\left(l_{0,t+1}\right) = \frac{y'_t}{l^o_t} - \frac{l^o_t l_{0,t}}{l^o_t}$$ or:

(45) $$bE_t\left(\frac{y'_{t+1} - l_{4,t+1}}{l^s_{t+1}/l^o_t}\right) - \frac{bE_t\left(l_{0,t+1}\right)}{1/l^o_t} = y'_t - l^o_t l_{0,t}$$

From (10), it follows:

(46) $$l^o_t = \frac{l^s_t}{1-q_t}$$

Introducing (46) into (45) gives:

(47) $$b \cdot E_t\left(\frac{y'_{t+1} - l_{4,t+1}}{(l^s_{t+1}/l^s_t)(1-q_t)}\right) - \frac{bE_t\left(l_{0,t+1}\right)}{(1/l^s_t)(1-q_t)} + \frac{l^s_t}{(1-q_t)} l_{0,t} = y'_t$$

Define:

(48) $$\frac{l^s_{t+1} - l^s_t}{l^s_t} = p^s_t$$

and introduce into (47):

(49) $$y'_t = b \cdot E_t\left(\frac{y'_{t+1} - l_{4,t+1}}{(1+p^s_t)(1-q_t)}\right) - \frac{bE_t\left(l_{0,t+1}\right)}{(p^s_t + 1)/l^s_{t+1} \cdot (1-q_t)} + \frac{l^s_{t+1}/(p^s_t + 1)}{(1-q_t)} l_{0,t}$$

Alternatively, define:

(50) $$J_t = \frac{l^o_t - l^s_t}{l^s_t}$$

(51) $$q_t = \frac{J_t}{1+J_t}$$

Insert (51) into (49) to get:

(52) $$y_t^{'} = bE_t\left(\frac{(y_{t+1}^{'} - l_{4,t+1})(1+J_t)}{(1+p_t^s)}\right) - \frac{bE_t(l_{0,t+1})(1+J_t)}{(p_t^s+1)/l_{t+1}^s} + \frac{l_{0,t}\cdot l_{t+1}^s(1+J_t)}{p_{t+1}^s}$$

Next, the condition for optimal holdings of legal cash can be written as:

(53) $$bE_t(l_{1,t+1}) = l_{1,t} + l_{5,t}$$

Applying (38) to the next period and inserting (43) yields:

(54) $$bE_t\left(\frac{y_{t+1}^{'}}{l_{t+1}^o} - l_{0,t+1}\right) = bE_t\left(\frac{y_{t+1}^{'} - l_{4,t+1}}{l_{t+1}^s}\right) - bE_t(l_{0,t+1}) + l_{5,t}$$

Applying (10) to the next period and rewriting gives:

(55) $$l_{t+1}^o = \frac{l_{t+1}^s}{1-q_{t+1}}$$

Inserting into (54) gives:

(56) $$bE_t\left(\frac{1-q_{t+1}}{l_{t+1}^s}y_{t+1}^{'} - l_{0,t+1}\right) = bE_t\left(\frac{y_{t+1}^{'} - l_{4,t+1}}{l_{t+1}^s}\right) - bE_t(l_{0,t+1}) + l_{5,t}$$

and finally, because $l_{t+1}^s = l_t^s(1+p_t^s)$, we get:

(57) $$bE_t\left(\frac{q_{t+1}}{1+p_t^s}y_{t+1}^{'} - l_{0,t+1}\right) = l_t^s(-l_{5,t}) + bE_t\left(\frac{l_{4,t+1}}{1+p_t^s}\right) - bE_t(l_{0,t+1})$$

As $bE_t(w+z) = bE_t(w) + bE_t(z)$, the term $-bE_t(l_{0,t+1})$ in equation (57) disappears!

Using (51) instead of (10), (57) becomes:

(58) $$bE_t\left(\frac{J_{t+1}}{(1+p_t^s)(1+J_{t+1})}y_{t+1}^{'}\right) = l_t^s(-l_{5,t}) + bE_t\left(\frac{l_{4,t+1}}{1+p_t^s}\right)$$

The left hand side of equation (57) can be interpreted as the expected production-weighted excess return to holding illicit cash rather than legal cash. This expression can either be positive or zero as the right-hand side can never turn negative taking into account that $l_{5,t}$ < 0. When the right-hand side is positive, legal cash is held for transactions' reasons, as the labor supply constraint on the parallel market is binding. If the left-hand side of equation (57) is zero, this implies that

$l_{5,t} = l_{4,t+1} = 0$. Thus, the labor supply constraint on the parallel market is not binding this time, but still some legal cash is held for other reasons than transactions in the legal labor market (legal cash as a long-term store of value). In a rather realistic world, firms will always hold some legal cash for hiring labor from the official market, otherwise the fiscal authorities will suspect production in the shadow economy. Moreover, even if they could hire all labor required from the (cheaper) parallel market, they will usually hold legal cash for other purposes. This is due to the fact that some cash belongs to the firms's portfolio according to the beliefs of shareholders and tax authorities. Taking a second-order Taylor approximation of equation (57) gives:

Proposition:

(59)
$$\begin{aligned} l_t^s\left(-l_{5,t}\right)+bE_t\left(\frac{l_{4,t+1}}{1+p_t^s}\right) &\cong b\left(\frac{E_t q_{t+1}}{1+E_t p_t^s}\right)y'\left(E_t i_{t+1}\right) \\ &+\frac{1}{2}b\,\frac{E_t\left(q_{t+1}\right)}{1+E_t p_t^s}\left[y'''\left(E_t i_{t+1}\right)\operatorname{var} i_{t+1}+\frac{2y'\left(E_t i_{t+1}\right)}{\left(1+E_t p_t^s\right)^2}\operatorname{var} p_t^s\right] \\ &+b\,\frac{1}{1+E_t p_t^s}\,y''\left(E_t i_{t+1}\right)\cdot\operatorname{cov}\left(i_{t+1},q_{t+1}\right) \\ &-b\,\frac{E_t q_{t+1}}{\left(1+E_t p_t^s\right)^2}\,y''\left(E_t i_{t+1}\right)\cdot\operatorname{cov}\left(i_{t+1},p_t^s\right) \\ &-b\,\frac{1}{\left(1+E_t p_t^s\right)^2}\,y'\left(E_t i_{t+1}\right)\cdot\operatorname{cov}\left(q_{t+1},p_t^s\right) \end{aligned}$$

The development of the this second order Taylor series with three variables follows the subsequent rule:

Proof:

(60)
$$\begin{aligned} E\left[f\left(i_{t+1},q_{t+1},p_t^s\right)\right] &\approx b_0+b_1 Var\left(i_{t+1}\right)+b_2 Var\left(q_{t+1}\right)+b_3 Var\left(p_t^s\right) \\ &+b_4 Cov\left(i_{t+1},q_{t+1}\right)+b_5 Cov\left(i_{t+1},p_t^s\right)+b_6 Cov\left(q_{t+1},p_t^s\right) \end{aligned}$$

We achieve the following parameter values:

(61)
$$b_0 = f\left[E\left(i_{t+1}\right),E\left(q_{t+1}\right),E\left(p_t^s\right)\right]=b\,\frac{E\left(q_{t+1}\right)}{1+E(p_t^s)}\,\frac{\partial y\left[E\left(i_{t+1}\right)\right]}{\partial i_{t+1}}$$

(62)
$$b_1=\frac{1}{2}\,\frac{\partial^2 f\left[E\left(i_{t+1}\right),E\left(q_{t+1}\right),E\left(p_t^s\right)\right]}{\partial {i_{t+1}}^2}=b\,\frac{E\left(q_{t+1}\right)}{1+E(p_t^s)}\,\frac{\partial^3 y\left(E\left(i_{t+1}\right)\right)}{\partial {i_{t+1}}^3}$$

(63) $$b_2 = \frac{1}{2}\frac{\partial^2 f\left[E(i_{t+1}),E(q_{t+1}),E(p_t^s)\right]}{\partial q_{t+1}{}^2} = 0$$

(64) $$b_3 = \frac{1}{2}\frac{\partial^2 f\left[E(i_{t+1}),E(q_{t+1}),E(p_t^s)\right]}{\partial p_t^{s\,2}} = 2b\,\frac{E(q_{t+1})}{\left[1+E(p_t^s)\right]^3}\frac{\partial y\left[E(i_{t+1})\right]}{\partial i_{t+1}}$$

(65) $$b_4 = \frac{\partial^2 f\left[E(i_{t+1}),E(q_{t+1}),E(p_t^s)\right]}{\partial i_{t+1}\partial q_{t+1}} = b\,\frac{1}{1+E(p_t^s)}\frac{\partial^2 y\left[E(i_{t+1})\right]}{\partial i_{t+1}{}^2}$$

(66) $$b_5 = \frac{\partial^2 f\left[E(i_{t+1}),E(q_{t+1}),E(p_t^s)\right]}{\partial i_{t+1}\partial p_t^s} = -b\,\frac{E(q_{t+1})}{\left[1+E(p_t^s)\right]^2}\frac{\partial^2 y\left[E(i_{t+1})\right]}{\partial i_{t+1}{}^2}$$

(67) $$b_6 = \frac{\partial^2 f\left[E(i_{t+1}),E(q_{t+1}),E(p_t^s)\right]}{\partial q_{t+1}\partial p_t^s} = -b\,\frac{1}{\left[1+E(p_t^s)\right]^2}\frac{\partial y\left[E(i_{t+1})\right]}{\partial i_{t+1}}$$

<u>q. e. d.</u>

Table 1 – KPSS Tests

Panel A - KPSS Statistics for the Null of Level Stationarity

	Truncation Lags			
Variable	0	1	2	3
m_t	3.43*	1.84*	1.29*	1.02*
y_t	3.73*	1.95*	1.34*	1.04*
w_t	3.70*	1.93*	1.33*	1.04*

Panel B - KPSS Statistics for the Null of Trend Stationarity

	Truncation Lags			
Variable	0	1	2	3
m_t	0.53*	0.31*	0.23*	0.19*
y_t	0.56*	0.30*	0.22*	0.18*
w_t	0.49*	0.26*	0.19*	0.15*

Note: m_t is real money balances of firms, y_t real gross value added of the business sector and w_t the wage wedge. * denotes significant statistics at the five per cent level where the critical values are from Kwiatkowski et al. (1992).

Table 2 – Cointegration Tests

Panel A - Johansen Procedure

Null	Trace	LM1	LM4	Estimated Cointegrating Vector
$r = 0$	36.62*	10.45	4.82	$m_t = 0.98y_t + 2.52w_t$
$r \leq 1$	14.78			
$r \leq 2$	1.19			

Panel B - Engle-Granger Approach

Cointegrating Regression:	$m_t = -3.10 + 1.01y_t + 1.97w_t + u_t$	
Cointegrating Tests:	CRDW = 1.32*	CRDF = – 4.71*

Note: m_t is real money balances of firms, y_t real gross value added of the business sector and w_t the wage wedge. The VAR has the lag length one and includes a constant term.
* denotes significant statistics at the five per cent level where critical values can be found in Engle and Yoo (1987), MacKinnon (1991) and Osterwald-Lenum (1992).