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# Is There a Policy of Optimal Wage and Income Differentiation? A Macroeconomic Perspective

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A Macroeconomic Perspective*

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## Contents

## Abstract

In this paper, we address the question of optimal wage and income dispersion in a growing economy. If already in the two-persons-case we have to deal with the fact of different marginal products of labor, there are two solutions in principle. Either two different wages are paid (at unanimous hours of working) according to these differences, or the good worker would have to work longer than the bad worker so as to equilibrate their marginal products (paying the same wage per hour). In the following parts of the paper, we limit our scope to the issue of wage/income, avoiding dealing with working hours differentiation. Furthermore, we argue on the macroeconomic level. Assuming a log-normal distribution of incomes, we determine the optimal degree of inequality and rate of per capita income growth from a model which integrates Okun's law with a social welfare function (with the Gini coefficient and the per capita growth rate as arguments) and a trade off relationship between equity and per capita growth. It is shown that modern industrial economies tend to have an "egalitarian bias" being most likely responsible for a considerable part of European unemployment.



Key words:

Income Distribution, Economic Growth, Labor Economics, Unemployment, Okun's law



Gibt es eine optimale Streuung von Löhnen und Einkommen in einer wachsenden Wirtschaft? Bereits im Zwei-Personen-Fall fordern abweichende Grenzproduktivitäten der Arbeit Lösungen, wobei entweder die Löhne bei gleichen Arbeitszeiten entsprechend differenziert werden oder der "bessere Arbeiter" bei unverändertem Einheitslohn umso viel länger arbeitet, bis es zu einem Ausgleich der Grenzproduktivitäten kommt. Auf makroökonomischer Ebene läßt sich eine optimale Schiefe der Einkommensverteilung begründen, wenn auf (1) eine soziale Wohlfahrtsfunktion, (2) auf Okun's Law und (3) eine log-normale Verteilung der Einkommen zurückgegriffen wird. Wir unterstellen dabei (zunächst abschnittsweise) eine Trade-Off-Beziehung zwischen der Pro-Kopf-Wachstumsrate des Einkommens und der Einkommensgleichverteilung. Es zeigt sich, daß demokratische Industriegesellschaften tendenziell einen "egalitären Bias" aufweisen, der mitverantwortlich für die hohe Unterbeschäftigung in Europa sein dürfte.



Schlagworte

Personelle Einkommensverteilung, Wirtschaftswachstum, Okun's Law, Arbeitsmarktökonomik

JEL-Klassifikation: J31, O40, D63, E24, O15

# 1 INTRODUCTION

Germany, France and many other European countries which have been striving so hard to complete the Maastricht criteria, are nowadays living a heated discussion in the field of wage and working hours policy. On the one hand, the wage formation process which is quite centralized and allows for little wage differentiation is severely criticized, being a hindering factor for higher employment and economic growth. On the other hand, french legislation has recently approved a law which fixes a maximum of 35 hours work per week in plants of more than 20 employees, while in Germany unions claim that a drastic reduction in the yearly overtime working hours would create massive additional emploment. It is Seldom understood that a uniformation of working hours has by and large the same effect as a reduction in wage differentiation.

There is an increasing amount of cross country empirical evidence pointing to the negative correlation between the change in the unemployment rate on the one hand and the change in the degree of wage dispersion on the other. ”Countries that have allowed relative wages of low-skilled workers to fall, have in general, seen the smallest increase in unemployment. Perhaps the best way to interpret the OECD’s recent study is that wage flexibility is a necessary, but probably not a sufficient condition for low unemployment” (The Economist 1996, p. 68). Of course, such correlations can be no substitute for a theoretical attempt to explain why (changes of) inequality of wages do matter when the employment issue is at stake. Moreover, it should be discussed just how far wage differentiation should be driven.

The more general issue deals with the question as to whether or not an overall higher dispersion of personal incomes can contribute to higher economic growth and to less unemployment. Surprisingly, the overwhelming majority of literature seems to recommend just the opposite as in the field of wage policy: Benabou (1996, 1996a) has put forward that (1996, p. 32) “in a more unequal society there is a greater political support for redistribution” (1996, p. 4) while at the same time “low inequality creates wide political support for efficient policies which prevent disparities from growing” (ibid, p. 7). This may or may not be true or not, but it is only meaningful when it is determined how far “disparities” are allowed to grow and how much efficiency is thought to be indispensable according to a social consensus and the like. According to Benabou (1996, 1996a), when capital and insurance markets are imperfect, redistributing wealth from richer to poorer agents can have a positive effect on economic growth because it relaxes the latter’s credit and/or liquidity constraints, reducing the number of people who do not have the collateral required to become entrepreneur, thus enhancing social mobility. Again, this may or may not be true, but it doesn‘t answer the question,of *how much* wealth should be redistributed!

There is an ”unpleasant logic” (equivalent to the ”unpleasant arithmetic” issue in monetary theory) which must be taken into account. Any unconditional statement claiming that less inequality (more growth) is conducive to higher growth (less inequality), cannot be correct. Why? Supposing it was true that less inequality persistently enhances growth, then only an equal distribution of incomes (in the strict sense) can lead to a maximum of growth. Nobody has any positive argument for such an axiom, nor can it be deducted implicitly from existing economic relationships, such as ”Kuznets' law”, etc. On the other hand, well performing countries - those with extraordinary high growth rates - do have additional resources for income redistribution. They will never - by the definition of a market economy along the lines of F. v. Hayek - have enough resources however, to eliminate every aspect of an existing skewness in income distribution. If we accept the logic of these arguments, only conditional statements on income distribution and growth can exist. The only meaningful yardstick for conditional statements is a benchmark or, likewise an „optimum“, of personal income distribution. An example: if actual inequality exceeds such an „optimum“, less inequality, i. e. a redistribution is helpful. Otherwise, if the actual situation (without an explicit target) serves as a „reference“, no (re)distribution policy is conceptually feasible. Hence, for any kind of rational (re)distribution policy, a benchmark or likewise „optimum“, must be presupposed. If such an „optimum“ would not exist in principle, the issue of (re)distribution of personal incomes policy becomes irrelevant. It is a different matter altogehter, whether multiple equilibria may or may not exist. It would come as no surprise, if due to changing social choices, the optimal (in)equality in a country varies over time.On the other hand, there is no reason for the assumption that „desired“ inequality is the same in different countries. On the contrary, one would expect a large variance of outcomes!

Against the background of this discussion; the purpose of this paper is threefold:

(i) first of all, we want to address the question, of how far wage/income differentiation should go from a social welfare maximizing point of view;

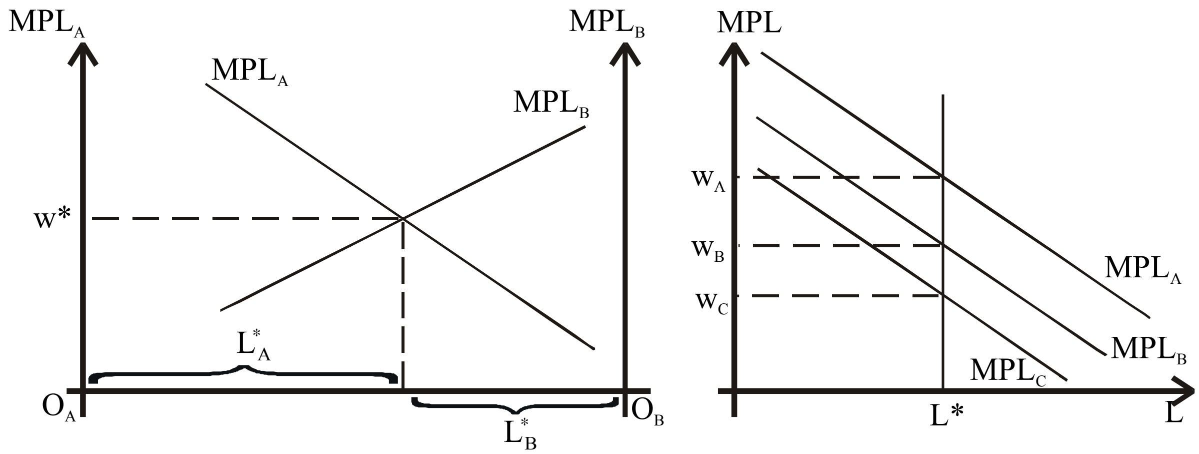
(ii) secondly, our interest is in the question, whether and how much, a certain degree of wage/income dispersion can contribute to optimize economic growth;

(iii) thirdly, we want to know what wage/income differentiation can do to optimize employment/to reduce unemployment.

## 2 OPTIMAL WAGE DISPERSION ON THE MICRO LEVEL

In a first simple, neoclassical approach, the intuition of the puzzle can be advanced. In Figure 1 (left part), we have depicted a box for the respective marginal productivity of labor curves of two individuals. At the intersection point we find the equilibrium wage rate $w^*$. The figure is drawn in such a way that $L_A + L_B$ is constant, making use of a standard tool in welfare economics or, likewise in the theory of international trade and based on the assumption of “other things being equal”. A change in the sum of hours worked can be introduced easily by enlarging or narrowing the angles of the box. All that matters is to think of a unit, say 100 % of working time, which can be distributed between two agents.

**Figure 1: Optimal Differentiation of Working Hours/Wages**




Source: Own compilation.

Clearly, at this unique equilibrium wage rate, different hours of labor are attributed to the two different agents. In general, individuals with higher (lower) marginal productivities of labor will have to work longer (shorter) than the average employee. On the other hand, firms may strive for uniform working hours. In such a situation (see right part of Figure 2), the difference in marginal productivities among workers (A, B, C for example) should be reflected in differentiated wage rates! In the following parts of the paper, we limit our scope to the issue of wage and not to working hours differentiation.

How far should wage differentiation go? "An equitable wage structure probably means that poorly qualified employees receive a higher wage than they deserve, corresponding to their marginal revenue product. Once they become unemployed, they will charge the social system, ask for common national funds built for public consumption purposes, and will thereby reduce the possibilities for investment of the public sector. On one hand, readiness of qualified persons to educational investment is reduced through an equitable income distribution because human capital is paid less interest. On the other hand, a very uneven personal income distribution with very low wages for the less qualified person, does not produce incentives to realize rationalization investments, nor to increase productivity. In a like manner, such a distribution of income lowers the possibility and the readiness to be successful. ... Hence, personal income distribution has the function of forcing to work as well as motivating to achieve good results. An extreme equality, as well as a marked inequality, is not advantageous and makes the necessary processes of adjustment and improvement during economic growth become more difficult."[1]

As a consequence the idea may be defended from a theoretical point of view, that an "optimal" degree of wage/income dispersion which is higher than it would be when having a uniform wage rate/unique income should exist, but still be lower than it would be in the case of total differentiation. This argument will be tackled more carefully in the next section, where we argue within a macroeconomic optimizing framework.

# 3 OPTIMAL WAGE (AND INCOME) DISPERSION ON THE MACRO LEVEL

Thus far, we have identified the determinants for optimal (efficiency maximizing) wage dispersion and employment on the micro level. *Macro*economic reflections on the distribution of personal incomes cannot simply be “derived” from, or directly based on, the foregoing microeconomic analysis and arguments. At first, there is a *differentiation problem.* Total income of the economy (national income) is equal to the sum of profits *and* wage incomes. Empirical and theoretical investigations (see for a survey Blümle 1989, pp. 21-25) suggest that profits - in a well functioning market economy driven by Schumpeterian forces of imitation and innovation - follow a log-normal distribution. According to our microeconomic insights, perfect wage differentiation can be achieved at unanimous hours of working. Hence, for the distribution of the log of wage income (wage level times hours worked in the economy or, alternatively the sum of log of wages and log of hours worked), only the dispersion of the wage rate matters for the optimality question rised above. For the empirical validity of a log-normal distribution of wages in the US, see Fortin/Lemieux (1997, pp. 83-86). We can take it as a stylized fact, that the abilities of people are distributed according to a log-normal distribution. In a sense, the distribution of US wage rates seems to adjust well to this fact.

1 Blümle/Sell (1998, pp. 333/34).

When it comes to the discussion of the distribution of all personal incomes, we also encounter an *aggregation* problem. As we know from statistics, only the product and not the sum of two variables, where each of the two follows a log-normal distribution, is also distributed according to a log-normal distribution (see Johnson/Kotz 1976, p. 119)! Also, private disposable income differs from national income by transfers payed to households and by direct income taxes collected from private households and firms. The government uses these instruments to modify personal income distribution as given by the market process and to introduce equity criteria into the distribution of private disposable incomes. Nevertheless, it is reasonable to assume the distribution of personal incomes to be skewed, and most likely left steep/lop-sided. Purely theoretical papers in our field, by the way, even “assume” “income distribution to be log-normal” (Glomm/Ravikumar 1992, p. 820).

What about personal income distribution and economic growth? The so-called ”new growth theories” have developed formulas of (constant) equilibrium per capita growth rates as a function of very few parameters. For instance, the famous Rebelo (1991) model finds the savings ratio s and the technology proxy A, as key parameters for the per capita economic growth rate. However, in all of these approaches there is an implicit assumption with regard to income distribution. The equilibrium per capita growth rate suggested, hinges on the existence of an incentive maximizing distribution of personal incomes!

Public choice contributions (Sachs 1989, Benhabib/Rustichini 1991, Perotti 1992, Perotti 1993, Alesina/Perotti 1994, Alesina/Rodrik 1994, Persson/Tabellini 1994, Alesina/Perotti 1996, Perotti 1996, Belletini 1998) and related studies (Falkinger/Zweimüller 1997) have stressed in recent years the negative impact of income inequality on social stability: ”The political consensus necessary for efficient growth may not be attainable if income inequality is too severe” (Benhabib/Rustichini 1991, p. 5). Weede (1997), however, has recently questionned the reliability of the data used in most of these papers. Robustness vanishes when data sets are adjusted (ibid). The literature interested in human capital formation (Glomm/Ravikumar 1992, Benabou 1996, 1996a, Chiu 1998) claims that redistribution relaxes credit/liquidity constraints for the education of the lower income groups and improves initial income distribution for future generations. These kind of models can also show that if ”inequality is sufficiently high, then the public education regime (in comparison to a private education regime, the authors) may yield higher per capita income for some future periods” (Glomm/Ravikumar 1992, p. 820). Chiu’s (1998, p. 45) very sophisticated analysis is able to demonstrate that ”if wealth is redistributed from rich to poor, aggregate human capital will increase as a result since the rich who are made poorer will stop sending their less talented sons to college and some of the poor who are more talented ... will find buying education justifiable as they are made richer. Then, to the extent that aggregate human capital determines growth or economic performance in general, greater income equality implies better economic performance” (ibid).

Both ”schools”, however, are not interested in solving the dilemma presented in our introduction. They do not say *how far* redistribution should go. Chiu’s analysis seems, in comparison, the closest to this requirement. However, instead of formulating a marginal decision criterion for policy makers with regard to the redistribution of wealth, Chiu simply argues with an ”exogeneous decrease in inequality” (ibid). Implicitly, it seems he is aware of a non-linear relationship: ”... more equal income distribution does not always imply better economic performance” (ibid, p. 46). In Blümle/Sell (1998), it is shown that not only theoretical considerations, but also empirical observations and estimations can substantiate a non-linear, concave relationship between per capita economic growth and (in) equality of income (ibid, pp. 347/8). Then, the puzzle can be resolved by making use of very simple analytical tools.

In the same vein, we postulate in the following, the existence of a strictly concave function which relates economic growth per capita (*y*) to a measure of personal income distribution, for instance the Gini coefficient[2]. As it is the case in equation (1), per capita economic growth is ”explained” by an autonomous component c (which could, for example, correspond to *sA* in the Rebelo model), where the society is penalized by growth losses whenever the actual personal income concentration (G) undergoes or surpasses the incentive maximizing income distribution, b. As a matter of fact, our approach is an extension of Breit’s “output possibilities curve” (1974, p. 12) where *output* is a concave function of the Gini coefficient[3]. Opposing Breit, we want to explore how the *growth* of the cake (instead of a given size) is traded against the equality of the slices (Lambert 1989, p. 110).

In order to be symetric, a quadratic form has to be given to the losses in terms of the per capita growth rate:

(1) $y = -a(G-b)^2 + c$ [4] *with* $a, b, c, \geq 0.$ $0 < b \leq 1$

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[2] The Gini coefficient has the advantage over the variance of log of incomes, not being restricted to a log-normal distribution, but rather to be used in most of the related empirical studies on the matter. Also, the Gini coefficient satisfies the Pigou-Dalton *and* the Bresciani-Turroni conditions, while the log-normal distribution violates the Pigou-Dalton condition. See Blümle (1975, p. 39-46).

[3] Making use of the Gini coefficient as a measure of (in)equality is not arbitrary. As Lambert (1989) and others have shown, the Gini coefficient is superior to other measures of inequality as it fulfills a number of important requirements for indices/conforms with principles of social choice, to become an argument of social welfare functions (Lambert 1989, pp. 112-129).

[4] We could complicate, or likewise enrich things further by letting c be dependent as well from G. The reasoning would stem from a Kaldorian/Pasinetti world in which the receivers of higher incomes tend to save more than the receivers of lower incomes!

From (1a) we may conclude that optimal solutions are only defined for a positive slope of the social indifference curve. The slope itself is endogenous as it decreases according to a higher Gini coefficient (a more unequal personal income distribution):

(1a) $$\frac{dy}{dG} = -2a[G - b] > 0 \;\; for\; G < b \quad with \quad \frac{d^2 y}{dG^2} = -2a < 0$$

Convex, upward sloping-indifference curves represent the fact, that per capita growth is a social "good" and inequality (as measured by the Gini coefficient) is a social "bad" (Lambert 1989, p. 110).

It is reasonable to assume that modern industrialized economies which are doing fairly well, do so because - among other things - they have deliberately chosen combinations of personal income distribution and per capita economic growth, which are prone to a stable and viable society. Hence, social welfare function does exist, deserving this classification (see Tinbergen 1963, p.6) [5], which assigns higher welfare values to a more equal income distribution at a given growth rate of per capita income and to higher growth rates of per capita income at a given distribution of personal income.[6]

(2) $$W = W(y,G) \; with \; W_y > 0 \; and \; W_G < 0$$

Notice that we have not given a specific functional form to the welfare function, nor have we introduced any weighting scheme to the different arguments (see for example Jenkins 1998, p. 27). Moreover, the specification of the welfare function corresponds to the insights of the capital asset pricing model in the theory of finance. S*ocial* welfare increases are associated with a higher "return" (with the per capita growth rate as a proxy) to the concerned society at a given "risk" for parts of the population of not *participating* in the higher output of the society (with the Gini coefficient as a measure for income concentration) or with a lower risk of non-participation[7] for parts of the population at a given "return" to the society as a whole[8]. From (2) we can take total derivatives and then deduct the marginal rate of substitution between per capita growth and the skewness of personal income distribution:

(2a) $$\frac{dy}{dG} = -\frac{\partial W}{\partial G} : \frac{\partial W}{\partial y} > 0$$

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[5] Boadway and Bruce (1984) argue instead (like Sen) using the mean income and a measure of (in)equality: „It would seem reasonable to assume that if one income distribution was unambigously more equally distributed than another according to the Lorenz criterion, and if mean income were also higher, then social welfare ought to be higher“ (ibid, p. 285).

[6] An obvious extension of our simple approach would consist in defining and maximizing a constrained intertemporal social welfare function for present and future generations!

[7] Notice that in development economics, „participation“ was an integral notion in the concept of „Integrated Rural Regional Development“ (IRRD) as well as a a synonym for the so-called „trickle down“ effects.

We can combine this approach with the following identity:

(3) $\Delta U \equiv U_t - U_{t-1}$

and a stylized version of Okun's law[9]:

(4) $\Delta U = d - e(y + \omega)$

where $\omega$ symbolizes population growth (assumed to be zero), $d$ stands for the change in the structural unemployment rate and $e$ is a proxy for the degree of capacity utilization. The maximisation of (2), constrained by the non-linear relationship given by (1) and a synthesis of (3) and (4) yields a corresponding Lagrange function which reads:

(5) $$L = W(y,G) + \lambda_1\left[y + a(G-b)^2 - c\right] + \lambda_2\left[U - d + ey - U_{-1}\right]$$

The optimality conditions then are:

(6a) $$\frac{\partial L}{\partial y} = \frac{\partial W}{\partial y} + \lambda_1 + e\lambda_2 = 0 \qquad \lambda_1 = -\frac{\partial W}{\partial y} - e\lambda_2$$

(6b) $$\frac{\partial L}{\partial G} = \frac{\partial W}{\partial G} + 2a\lambda_1[G - b] = 0 \Rightarrow \frac{\partial W}{\partial G} = [b - G]2a\lambda_1 = \begin{cases} > 0 \ \ for\ G > b \\ < 0 \ \ for\ G < b \end{cases}$$

The first alternative in the solution of (6b) however, is not a maximum, given the requirements of the welfare function (see below)!

(6c) $$\frac{\partial L}{\partial \lambda_1} = y + a(G-b)^2 - c$$

(6d) $$\frac{\partial L}{\partial \lambda_2} = U - d + ey - U_{-1}$$

Rearanging terms and solving for optimal solutions, we achieve:

(6e) $$G^* = b + \frac{\partial W / \partial G}{(\partial W / \partial y + e\lambda_2)} \frac{1}{2a} < b \ \ for \begin{cases} \lambda_2 > 0 \ or \\ \lambda_2 < 0, but\ |e\lambda_2| < |\partial W / \partial y| \end{cases}$$

(6f) $$y^* = c - \frac{1}{4a}\left\{\frac{\partial W / \partial G}{(\partial W / \partial y + e\lambda_2)}\right\}^2 < c \ \ for \begin{cases} \lambda_2 > 0 \ or \\ \lambda_2 < 0, but\ |e\lambda_2| < |\partial W / \partial y| \end{cases}$$

8 In public finance, the growth rate of the real economy is compared with the real interest rate on public debt in order to evaluate the sustainability of the latter! Within a social welfare function, such a rate of return has to be „normalized“ by taking into account population growth.

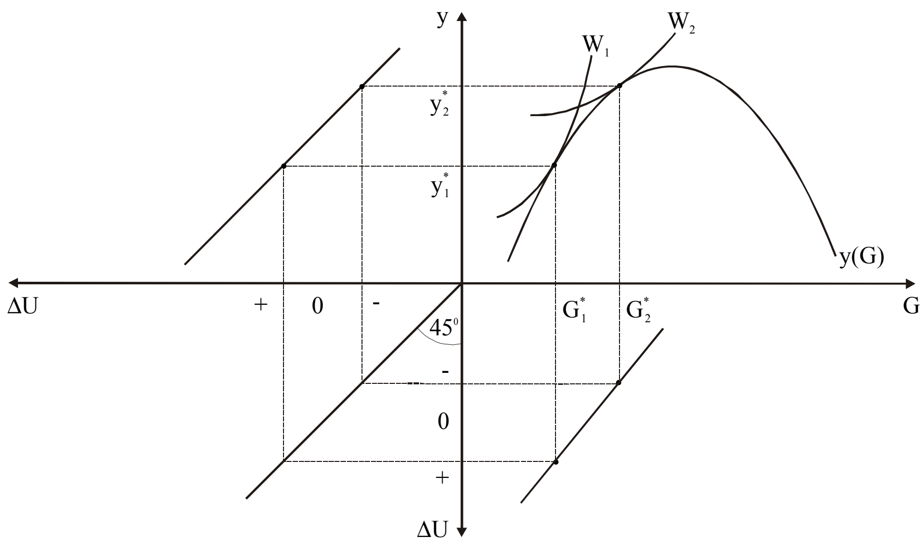
9 See Klös 1992. Obviously, a combination of (3) and (4) leads to a first order difference equation for $U_t$ which can be solved. We disregard this aspect in the following without losing generality in our results!

(6g) $U^{*} = d - ey^{*} + U_{-1} > d - ec + U_{-1}$

The optimal dispersion of incomes should be smaller than the incentive maximising, b: solutions to the right of b are inferior to solutions to the left of b. The conditions applicable here are listed in (6e) and, all the more, they emanate from the welfare function itself. As shown in the first quadrant of Figure 2, any pair of per capita income growth rate/Gini coefficient which has the same per capita growth, but a more equal income distribution, dominates vis-a-vis another pair with a more unequal income distribution (see Lambert 1989, pp. 123-125) at the same per capita growth rate. The result for the optimal dispersion of incomes as given by equation (6e) stands for a sort of „egalitarian bias“[10] in modern democracies which has its equivalent in the „inflationary bias“ so often found in models of political business cycles. On the other hand, optimal growth (optimal unemployment rate) is lower (higher) than the one put forward by new growth theories, because the latter normally disregard distributional issues.

In Figure 2, we have depicted a straightforward graphical interpretation of the analytical results. In the first quadrant we have drawn the non-linear, concave relationship between inequality and per capita growth. Two alternative social welfare functions $(W_1, W_2)$ lead to different tangential solutions. Simplifying, one can say that an egalitarian society achieves a lower rate of per capita growth, while a society which reaches a high per capita growth rate, gives in with regard to its equity goals.

**Figure 2: (In)Equality, Economic Growth and Change in Unemployment**



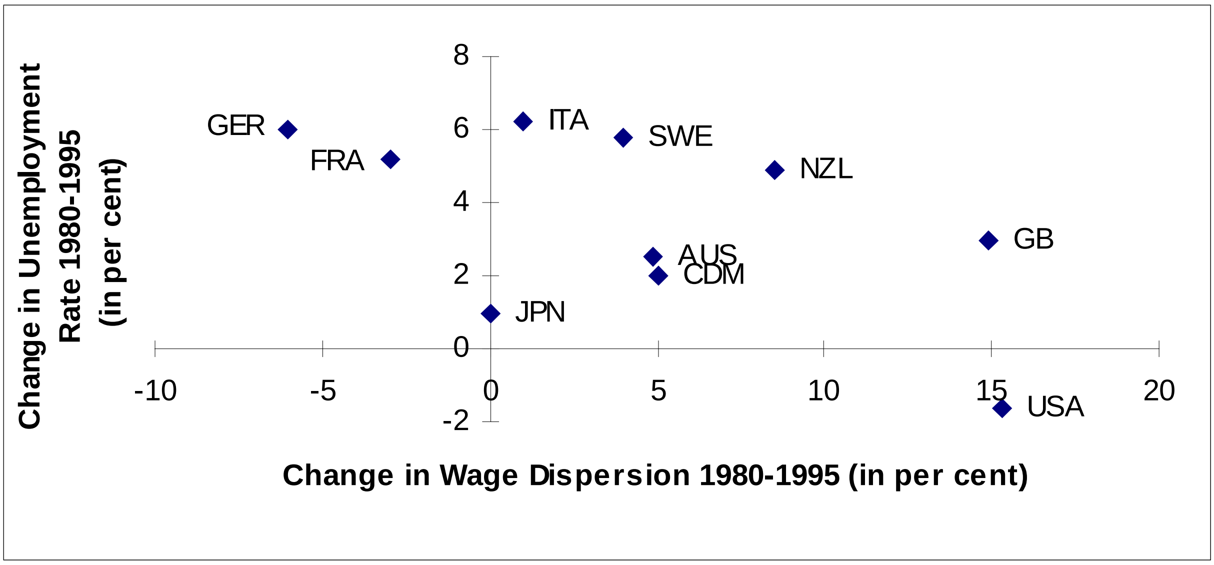

Source: Own compilation.

[10] Sweden is an impressive example for this effect: a country ranked third among the OECD countries in terms of his GDP per capita in the 1960s (before the welfare state experiment was undertaken), it is nowadys on position 17 or so. See also Blümle/Sell 1998, p. 342.

The second quadrant contains Okun's law according to equation (4). Of course, the position of this line can change with time, or differ between various countries. The flatter the line, the easier it is to reduce unemployment in an economy, ceteris paribus. With the help of the 45 degree line in the third quadrant, we can identify changes in the unemployment rate associated with different positions in the distribution of personal incomes in quadrant four.

The following Figure 3 reproduces a graph from the Economist of 1996, in which selected country experiences with (change in) unemployment and (change of) wage dispersion were collected. It would appear that the stylized facts from Figure 3 are nicely explained by our simple macro model from above (Figure 2). Countries which allowed for a significant positive change in wage dispersion (hence a higher Gini coefficient G, provided that the distribution of non-wage incomes did not change in the opposite direction) could achieve a reduction (or negative change) in their unemploment rate.

**Figure 3: Wage Dispersion and Unemployment in Selected Countries (1980-1995)**




Source: The Economist of 08/18/1996, p. 68.

Our achieved results however, still suffer from the following shortcomings. *Firstly,* we have not considered a Gini coefficient for a lognormal distribution. *Secondly*, even if we assume constant values of the partial derivatives, we may run into problems with the second-order conditions. *Thirdly*, our first order conditions and hence optimal solutions, still depend on partial derivatives of the welfare function, which in turn depend on the optimal values, so that (6a) through (6d), or likewise (6e), through (6g), is a set of implicit equations and we still miss explicit solutions.

Let us discuss these problems one by one; the *first issue* can be resolved easily. The Gini coefficient for a lognormal distribution is given by (see Chotikapanich/Valenzuela/Prasada Rao 1998, p. 68):

$$(7) \qquad G = 2\Phi\left(\frac{\sigma_x}{\sqrt{2}}\right) - 1$$

where $\sigma_x$ is the standard deviation of a lognormal distribution and $\Phi$ is the standard normal integral up to the ordinate $\frac{\sigma_x}{\sqrt{2}}$ (ibid). The Lagrangean now reads:

$$(8) \qquad L = W(y, 2\Phi\left(\frac{\sigma_x}{\sqrt{2}}\right) - 1) + \lambda_1\left[y + a\left[(2\Phi\left(\frac{\sigma_x}{\sqrt{2}}\right) - 1) - b\right]^2 - c\right] + \lambda_2[U - d + ey - U_{-1}]$$

so that:

$$(9) \qquad \frac{\partial L}{\partial \sigma_x} = \frac{\partial W}{\partial G}\frac{2\Phi}{\sqrt{2}} + 2a\lambda_1\left[2\Phi\frac{\sigma_x}{\sqrt{2}} - 1 - b\right]\frac{2\Phi}{\sqrt{2}} = 0 \Rightarrow \sigma_x^* = \frac{\sqrt{2}}{2\Phi}\left[(1+b) - \frac{1}{2a\lambda_1}\frac{\partial W}{\partial G}\right]$$

With regard to our *second issue,* we can say that the first second order condition is fulfilled if:

$$(10) \qquad \frac{\partial^2 L}{\partial \sigma_x^2} = 4a\lambda_1\Phi\sqrt{2}^{-1} < 0 \quad \textit{for } \lambda_1 < 0$$

If $\lambda_2$ is positive, (10) is always met. If $\lambda_2$ is negative, the achievement of the first second order condition depends on whether $\frac{\partial W}{\partial y} > e\lambda_2$. But, as we have shown above, the requirements of the welfare function do secure both of these conditions. The next first order condition is:

$$(11) \qquad \frac{\partial L}{\partial y} = \frac{\partial W}{\partial y} + \lambda_1 + e\lambda_2 = 0 \qquad \lambda_1 = -\frac{\partial W}{\partial y} - e\lambda_2$$

Finally, the second order condition reads:

$$(12) \qquad \frac{\partial^2 L}{\partial y^2} = \frac{\partial^2 W}{\partial y^2} < 0$$

and can be taken to be fulfilled if the welfare function is “well behaved”. The third and fourth first order conditions simply reproduce the two restrictions of the optimization (see above). They can be used to compute the optimal per capita growth rate and the optimal rate of unemployment:

(13) $$y^* = c - a\left\{(2\Phi\left(\frac{\frac{\sqrt{2}(1+b)}{2\Phi} - \frac{1}{2a\lambda_1}\frac{\partial W}{\partial G}}{\sqrt{2}}\right) - 1) - b\right\}^2 = c - a\left[\frac{\Phi}{a\lambda_1\sqrt{2}}\frac{\partial W}{\partial G}\right]^2 < c$$

(14) $$U^* = d - e\left\{c - a\left[\frac{\Phi}{a\lambda_1\sqrt{2}}\frac{\partial W}{\partial G}\right]^2\right\} + U_{-1} > d - ec + U_{-1}$$

It only remains now to discuss our *third problem* from above. Now as it stands, the optimal skewness of income distribution (9), as well as the optimal per capita growth rate (13) and the optimal rate of unemployment (14) are not *explicit solutions* as they still depend on $\lambda_1$ and $\lambda_2$. In order to overcome this deficit, we give a certain specification to our social welfare function, as similarly, a semi-logarithmic specification from the Weber-Fechner type:

(15) $$\mathrm{W} = \gamma \ln \mathrm{y} + (1-\gamma)\ln(1-\mathrm{G})$$

Then we have the following first order conditions:

(16) $$\frac{\partial \mathrm{W}}{\partial \mathrm{y}} = \gamma \frac{1}{\mathrm{y}}$$

(17) $$\frac{\partial \mathrm{W}}{\partial \mathrm{G}} = -(1-\gamma)\frac{1}{1-\mathrm{G}}$$

Introducing (17) into (9) leads to:

(9a) $$\sigma_x = \frac{1}{\sqrt{2}\Phi}\left(1 + \mathrm{b} + \frac{1-\gamma}{2\mathrm{a}\lambda_1\left(2-\sqrt{2}\Phi\sigma_x\right)}\right)$$

Making use of (16) we can rewrite (11):

(11a) $$\lambda_1 = -\frac{\gamma}{\mathrm{y}} - \mathrm{e}\lambda_2$$

Equations (9a), (11a) in conjunction with the third (18)and fourth (19) first order conditions,

(18) $$\mathrm{y} = \mathrm{c} - \mathrm{a}\left[2\Phi\left(\frac{\sigma_x}{\sqrt{2}}\right) - 1 - \mathrm{b}\right]^2$$

(19) $$\mathrm{U} = \mathrm{d} - \mathrm{ey} + \mathrm{U}_{-1}$$

are a set of four equations with five unknown variables: U, y, $\lambda_1$, $\lambda_2$ and $\sigma_x$. Hence, one equation is „missing“! The problem can be solved if we weigh social welfare, as given by equation (15), with (1-U) the rate of employment:

(15a) $$\mathrm{W} = \gamma \ln \mathrm{y} + (1-\gamma)\ln(1-\mathrm{G}) + \ln(1-\mathrm{U})$$

The rate of employment is a good proxy for the degree of human capital usage in a society. As modern growth theories argue, human capital and its positive external effects belong at the same time, to the main forces which drive economic growth (see Barro and Sala-i-Martin 1995). Accordingly we achieve an additional first order condition:

(20) $$\frac{\partial L}{\partial U} = \frac{\partial W}{\partial U} + \lambda_2 ; \frac{\partial W}{\partial U} = \frac{1}{1-U}(-1)$$

Making use of (15a) we can substitute $\partial W / \partial U$ and solve for $\lambda_2$:

(21) $$\lambda_2 = \frac{1}{1-U}$$

We now have a simultaneous system of five equations with five unknown variables, which has a unique solution. Unfortunately, due to the occurence of polynoms of higher order, it is not possible to present it in an explicit form, but we can make a simple case whenever we assume $c = 0$. Then we achieve:[11]

(22) $$y = 0$$

(23) $$\sigma_x = 1 + b$$

(24) $$U = d + U_{-1}$$

(25) $$\lambda_2 = \frac{1}{1-(d + U_{-1})}$$

(26) $$\lambda_1 \rightarrow \infty$$

In economic terms, this scenario corresponds to a neoclasical world with a steady state per capita growth rate of zero. In Figure 2, such a constellation would shift the whole parabola from quadrant I into quadrant IV, the maximium of the curve becoming tangential to the G-axis! Notice that in such a world any type of redistribution departing from the incentive maximising value b, leads to negative per capita growth rates. Hence, as long as there are no significant amounts of an accumulative factor of production (as human capital in the new growth theories) which secures the existence of a positive steady state per capita growth rate, any sort of redistributive policy has desastrous effects.

[11] See mathematical annex.

# 4 SUMMARY AND POLICY RECOMMENDATIONS

What does all this mean for economic policy? Our results, though of macroeconomic nature, confirm – according to the subsidiarity principle - the conviction that wage contracts between firms and employees should not be ”corrected” by wage legislation of the government. If the society as a whole is not satisfied with the outcome of the market and the wage bargaining processes, direct (positive and negative) transfers are the appropriate instruments to accomplish higher equity requirements. A policy of income redistribution presupposes the existence of a significant amount of accumulative factors in an economy, otherwise negative per capita growth rates may easily arise. In other words: only an advanced economy is a candidate for redistribution. However, as we have seen, the modern welfare state tends to produce an “egalitarian bias” which causes not only losses in terms of a lower per capita growth rate, but also in terms of a higher unemployment rate. Perhaps, traditional concepts of “justice and equity” are somehow misled. Instead of being so concerned with the *results* in a market economy, which should by and large reflect the different talents and capabilities of the individuals, more attention should almost surely be paid to an equitable distribution of *access* to well developed education and formation institutions.

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## MATHEMATICAL ANNEX

**Proposition 1**: For $c = 0$, it follows $y = 0$.

**Proof**:

Introducing (9a) into (18), assuming $c = 0$, yields:

$$\text{(A1)} \qquad y = -a\left(\frac{1-\gamma}{2a\lambda_1\left(2-\sqrt{2}\Phi\sigma_x\right)}\right)^2$$

Next, using (21) and (19), we can rewrite (11a):

$$\text{(A2)} \qquad \lambda_1 = -\frac{\gamma}{y} - e\frac{1}{1-d+ey-U_{-1}} = -\frac{\gamma\left(1-d+ey-U_{-1}\right)+ey}{y\left(1-d+ey-U_{-1}\right)}$$

Substituting $\lambda_1$ in (A1) yields:

$$\text{(A3)} \qquad y = -a\left(-\frac{(1-\gamma)y\left(1-d+ey-U_{-1}\right)}{2a\left[\gamma\left(1-d+ey-U_{-1}\right)+ey\right]\left(2-\sqrt{2}\Phi\sigma_x\right)}\right)^2$$

or

$$\text{(A3a)} \qquad y = -ay^2\left(-\frac{(1-\gamma)\left(1-d+ey-U_{-1}\right)}{2a\left[\gamma\left(1-d+ey-U_{-1}\right)+ey\right]\left(2-\sqrt{2}\Phi\sigma_x\right)}\right)^2$$

The only non-negative solution for this expression is $y = 0$; This value of $y$ immediately leads to $U = d + U_{-1}$ and (because $\lambda_1 \to \infty$) to $\sigma_x = 1 + b$.

**<u>Proposition 2</u>**: From *c*, *y = 0*, it follows $G = b$ and $\Phi = \frac{\sqrt{2}(b+1)}{2(1+b)} = \frac{1}{2}\sqrt{2}$

**<u>Proof</u>**: The first part of proposition 2 is straightforward from (1). For the second part of proposition remember that equation (7) stated that:

(A4) $$G = 2\Phi\left(\frac{\sigma_x}{\sqrt{2}}\right) - 1$$

Introducing (23) into (7) gives:

$$G = 2\Phi\left(\frac{1+b}{\sqrt{2}}\right) - 1 \overset{!}{=} b$$

(A5) $$2\Phi(1+b) = \sqrt{2}(b+1)$$

$$\Phi = \frac{\sqrt{2}(b+1)}{2(1+b)} = \frac{1}{2}\sqrt{2}$$

q.e.d.