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Implications of Salience Theory: Does the Independence Axiom always hold under Uncertainty?

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Implications of Salience Theory:
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Abstract

So far, "salience theory of choice under risk" has been mainly applied to situations of risk rather than to those of uncertainty. In this paper, we show that salience theory provides the prediction that Allais paradoxes should never occur in the context of uncertainty. A finding which contradicts the scarce evidence existing up to now, and which indicates that further research on the topic is necessary.

Keywords: ■ Models of decision-making under risk and uncertainty ■ Allais paradox ■ Independence axiom ■ Salience theory

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1. Introduction

In the course of the various documented anomalies in choice under risk and uncertainty, i.e. the violations of expected utility theory, several descriptive theories have been developed. A relatively young and promising model is "salience theory of choice under risk" (Bordalo et al., 2012). Its framework is able, inter alia, to reproduce the fourfold pattern of risk attitudes and preference reversals. While also accounting for the various examples of the Allais paradox, the model makes the strong prediction that the independence axiom should hold when the lotteries to choose from are correlated – a finding which was empirically confirmed by Frydman & Mormann (2017). However, this prediction has so far only been considered in the field of risk. In fact, as we show in this paper, in the context of uncertainty, the – in such a case unavoidable – correlation between lotteries would prevent Allais paradoxes from the outset according to the model. We present an unincentivized survey of our own that supports this proposition, in contrast to so far existing empirical evidence on the topic. We argue that further empirical research is desirable, as until now, Allais paradoxes under uncertainty have been unchallenged, yet investigated less thoroughly than those under risk.

2. Salience theory's mode of operation

This section briefly summarizes the model of salience theory (a more detailed presentation is given in Bordalo et al., 2012). The description of a choice problem is such that, from the set of all possible states of the world, S, a state $s \in S$ is defined with known and objective probability π_s where $\sum_{s \in S} \pi_s = 1$ and a lottery L_i disburses payoff x_s^i in state s (we only refer to situations where two lotteries L_i and L_j are offered for selection). A decision maker's total evaluation of a lottery depends on the combination of a probability weighting and a value function, where the latter is mostly assumed to be linear. The decision maker is a "Local Thinker", which means she overweights states that draw her attention. Probabilities are distorted in the following way: First, a salience ranking is made among all states. Second, depending on a state's salience, its probability is overweighted or underweighted in the evaluation process of L_i, with a higher salience rank leading to a higher distorted probability. A state's salience is determined by a salience function, which is a continuous and bounded function satisfying three conditions: Ordering, diminishing sensitivity and reflection. The salience function $\sigma(x_s^i, x_s^j)$ for state s generates higher values, the more salient s is, which applies – in simple terms – the more the payoffs in s differ and the closer the average of these payoffs gets to zero. The final evaluation of

a lottery, hence, presents as follows (where ω_s^i is the decision weight generated in the course of the salience ranking, and v is the value function):

$$V^{LT}(L_i) = \sum_{s \in S} \pi_s \cdot \omega_s^i \cdot v(x_s^i)$$

The functioning of the model can be demonstrated particularly well with the example of an Allais paradox given by Kahneman & Tversky (1979). Participants were presented the following choice problem, with outcomes referring to Israeli currency:

Problem 1: Choose between $L_1(z)$ and $L_2(z)$

$$L_1(z) = \begin{cases} 2500 \text{ with prob. } 0.33 \\ 0 \quad\quad\quad\quad 0.01 \\ z \quad\quad\quad\quad 0.66 \end{cases} \qquad L_2(z) = \begin{cases} 2400 \text{ with prob. } 0.34 \\ z \quad\quad\quad\quad 0.66 \end{cases}$$

In one version of the problem, the common consequence was $z = 2400$ which lead 82% of 72 subjects to be risk-averse and choosing L_2, while in another version, where $z = 0$, a similar share of the respondents was risk seeking, preferring L_1 over L_2. 61% of all subjects made the modal choice in both problems – thereby violating the independence axiom. Salience theory can account for this observation in the following way: The state space S is the "product space induced by the lotteries' marginal distributions over payoffs" (Bordalo et al., 2012: p.1257), i.e. all possible combinations of payoffs of the two lotteries. For $z = 2400$, the state space is $S = \{(2500, 2400), (0, 2400), (2400, 2400)\}$, whereas the subsequent salience ranking is $\sigma(0, 2400) > \sigma(2500, 2400) > \sigma(2400, 2400)$. As a result, the state $(0, 2400)$, where lottery L_1 pays nothing, is overweighted and hence makes the lottery less attractive. This explains the majoritarian choice pattern in favor of L_2. In contrast, when $z = 0$, the state space turns out to be $S = \{(2500, 2400), (2500, 0), (0, 2400), (0,0)\}$, with the corresponding salience ranking $\sigma(2500, 0) > \sigma(0, 2400) > \sigma(2500, 2400) > \sigma(0, 0)$. In this case, the state $(2500, 0)$ – where L_1 disburses its maximum payoff, while L_2 pays nothing – is most salient, explaining the risk seeking choice in favor of L_1, observed by most subjects.

3. The absence of Allais paradoxes in the course of correlated lotteries

Crucial for salience theory's explanation of the Allais paradox observed in choice problem 1 is that the decision maker sees the lotteries as independent, i.e. the state space equals all possible combinations of outcomes. When choice problem 1 is presented in a correlated version as in 1', the state space perceived by the Local Thinker automatically changes or rather reduces.

Problem 1': Choose between $L_1(z)$ and $L_2(z)$

Probability	0.01	0.33	0.66
payoff $L_1(z)$	0	2500	z
payoff $L_2(z)$	2400	2400	z

According to salience theory, the decision maker now faces the following, altered state space: $S = \{(z, z), (2500{,}2400), (0{,}2400)\}$. The correlation prevents L_1's payoff z from occurring in a state together with any outcome from L_2 other than z – and vice versa. The corresponding salience ranking is $\sigma(0, 2400) > \sigma(2500{,}2400) > \sigma(z, z)$, where the state (z, z) obtains the lowest level of salience, no matter what value z takes on. The invariant salience ranking entails that also the probability distortion of all three states is fixed. In consequence, any value of z enters both lotteries' evaluation with the same probability weighting. The independence axiom now holds according to salience theory, so that the switch in risk attitudes exhibited by the majority of subjects in problem 1 should vanish. And in fact, in the empirical analysis of Bordalo et al. (2012), 80% of subjects did not violate the independence axiom, with a great majority being risk-averse in both problems as predicted by the model. This finding is confirmed by a more elaborate empirical analysis from Frydman & Mormann (2017: p.3), who identify that "correlation does systematically impact risk taking in the manner predicted by Bordalo et al., as the propensity to exhibit the Allais paradox monotonically decreases in the correlation between lotteries".

4. Choice under uncertainty – an inevitable correlation

Up to now, salience theory has been basically applied to risk. However, the fundamental psychological supposition of salience and local thinking should naturally be valid for the context of uncertainty as well. Considering the specific representation of choice under uncertainty, it

becomes clear that lotteries offered in choice problems in the field of uncertainty will always be correlated.

In the context of uncertainty, probabilities are unknown, and payoffs are disbursed in dependence of the occurrence of certain events whose likeliness can only be estimated. As a result, when testing the independence axiom, the common consequence added to both lotteries must depend on the same event to ensure that z is indeed considered an identical add-on. This inherent correlation would lead salience theory to suggest that an Allais paradox should never be the majoritarian choice pattern in the context of uncertainty – a claim that is opposing empirical findings from e.g. Tversky & Kahneman (1992) and Wu & Gonzalez (1999). Tversky & Fox (1995) even find support for subadditivity being more pronounced for uncertainty than for risk, which would make Allais Paradoxes even more likely in the field of uncertainty. However, compared to the field of risk, the empirical evidence on Allais paradoxes under uncertainty is rather moderate.

In this regard, we conducted a preliminary, unincentivized empirical analysis of our own in December 2017 via Google Forms. The survey involved 121 participants, composed of students, workers and retirees – both male and female. In our analysis, we tested several propositions and found strong support for e.g. the fourfold pattern of risk attitudes, Allais paradoxes in choice under risk and the disappearance of these Allais paradoxes when lotteries were presented in a correlated form. In addition, the subjects in our survey were confronted with choice problem 2 (with outcomes denominated in euro) which tests for the independence axiom under uncertainty:

> **Problem 2:** Choose between lotteries L_1 and L_2, which disburse different payoffs in dependence of the temperature on Christmas Eve 2017 at 6:00 p.m. in Munich. The table below displays which amount of money each lottery pays in dependence of the ranges of temperature A, B and C.

	A Temperature below –2°C	**B** Temperature between –2° and 1°C	**C** Temperature above 1°C
Lottery L_1	250	z	250
Lottery L_2	750	z	0

The choice problem was tested twice, with the common consequence z, once being zero and once being 250. To make sure that the two versions were answered independently of one another, they were placed far apart from each other in a series of eleven different choice problems. A vast majority of 75%, which is statistically significant at the 0.01 level, did not exhibit an Allais paradox (with 58% of subjects choosing L_1 in both problems and 17% of subjects choosing L_2 in both problems). Therefore, the results give support to the hypothesis of salience theory that the state space determines the salience of payoffs, which in turn shapes a decision maker's risk attitudes.

5. Conclusions

The great explanatory power of salience theory in the context of choice under risk has been verified in several empirical investigations. What so far has been neglected, is an application of the model in the field of uncertainty. In this paper, we show that – next to its implication that Allais paradoxes should not occur in the context of risk when lotteries are correlated – salience theory makes the even stronger prediction that under uncertainty, Allais paradoxes should not occur at all. A conclusion, which contradicts hitherto existing empirical findings. However, this empirical evidence is rather limited, compared to the comprehensive research in the field of risk. Therefore, we propose additional empirical research on this topic as with salience theory, a conclusive theory predicts the opposite of what so far has been the general perception. In fact, our own unincentivized survey supports salience theory's prognosis, as the independence axiom holds under uncertainty for 75% of 121 subjects in the respective test. A possibility for further research could be to design uncertain choice problems similar to the classical risky – Allais paradox inducing – choice problems, in which the likelihood of the uncertain events approximately corresponds to the respective probabilities of the risky choice problems.

Two different scenarios are possible: Either the common consequence effect also applies to uncertainty, and salience theory suffers a serious setback as its notion of how the state space shapes attitudes toward risk is disproved. Or, as violations of the independence axiom were so far considered a common element of choice under uncertainty, this chapter might need to be rewritten.

References:

Bordalo, P., Gennaioli, N. and Shleifer, A. 2012. Salience Theory of Choice under Risk. *The Quarterly Journal of Economics.* **127** (3), pp. 1243-1285.

Frydman, C., Mormann, M. 2017. The role of Salience and Attention in Choice under Risk: An experimental Investigation.

Kahneman, D. and Tversky, A. 1979. Prospect Theory: An Analysis of Decision under Risk. *Econometrica*. **47** (2), pp. 263-291.

Tversky, A. and Fox, C. 1995. Weighing Risk and Uncertainty. *Psychological Review.* **102** (2), pp. 297-323.

Tversky, A. and Kahneman, D. 1992. Advances in Prospect Theory: Cumulative Representation of Uncertainty. *Journal of Risk and Uncertainty.* **5** (4), pp. 297-323.

Wu, G. and Gonzalez, R. 1999. Nonlinear Decision Weights in Choice Under Uncertainty. *Management Science.* **45** (1), pp. 74-85.

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